Mini-Lecture 3.1
Simplifying Algebraic Expressions

Learning Objectives:

1. Use properties of numbers to combine like terms.
2. Use properties of numbers to multiply expressions.
3. Simplify expressions by multiplying and then combining like terms.
4. Find the perimeter and area of figures.
5. Key Vocabulary: algebraic expression, constant, variable, numerical coefficient, like terms, distributive property, and simplify.

Examples:

1. Simplify each expression by combining like terms.
   a) \(3x + 5x\)  
   b) \(11y - 9y\)  
   c) \(5a - 19a\)  
   d) \(6z + 15z - 5z + 7\)  
   e) \(4.2 + 8.7x - 1.9 - 3.3x\)  
   f) \(\frac{4}{5}x - \frac{2}{3} + \frac{1}{3}x - \frac{1}{5}\)

2. Multiply.
   a) \(7(4x)\)  
   b) \(-12(6a)\)  
   c) \(\frac{2}{5}(-15x)\)  
   d) \(-5(3y - 2)\)

3. Simplify each expression. Use the distributive property to remove parentheses.
   a) \(5(y + 3) - 6\)  
   b) \(2(7 - 3a) + a\)  
   c) \(-3(x + 1) + 4(8 - x) - 18\)

4. Find the perimeter of the figures.
   a) \(6\) in \(5\) in \(5\) in \(6\) in
   b) \(12\) ft

Teaching Notes:

- Some students need to practice identifying “like terms”.
- Some students do not know that a variable without a numerical coefficient actually has a coefficient of 1.
- Many students tend to make careless errors associated with the distributive property.
- Remind students that perimeter is the distance around an object.

Answers: 1a) 8x, b) 2y, c) -14a, d) 16z + 7, e) 5.4x + 2.3, f) \(\frac{17/15}{13/15}\) 2a) 28x, b) -72a, c) -6x, d) -15y + 10; 3a) 5y + 9, b) 14 - 3a, c) -7x + 11; 4a) 16m, b) 36 ft
Mini-Lecture 3.2
Solving Equations: Review of the addition and multiplication properties

Learning Objectives:

1. Use the addition property or the multiplication property to solve equations.
2. Use both properties to solve equations.
3. Translate word phrases to mathematical expressions.
4. Key Vocabulary: expression, and equation

Examples:

1. Solve. Check each solution.
   
   a) \(-14 = 15 + x\)  
   b) \(3y - 7y = 12\)  
   c) \(\frac{x}{4} = 11 - 5\)  
   d) \(5x + 2 - 4x = 7 - 19\)  
   e) \(3(3x - 5) = 10x\)  
   f) \(13x = 4(3x - 1)\)

2. Solve. Both properties.
   
   a) \(5y + 2 = 17\)  
   b) \(3x - 5 = 10\)  
   c) \(-4(x + 2) - 60 = -8\)  
   d) \(9 - 14 = \frac{x}{-12}\)

3. Write each phrase as a variable expression. Use \(x\) to represent “a number.”
   
   a) Eight subtracted from a number
   b) The product of a number and 5
   c) The quotient of a number and negative 7
   d) The total of twice a number and 3

Teaching Notes:

- Encourage students to write down all steps in a neat, organized manner. This habit will help students as equations increase in difficulty.
- Encourage students to use the addition property in such a way that the variable ends up with a positive coefficient.
- Mention to students that it does not matter on which side of the equation you isolate the variable.
- Remind students to always check their final answer by substituting it back into the original equation.

Answers: 1a) -29, b) -3, c) 24, d) -14, e) -15, f) -4; 2a) 3, b) 152, c) -15, d) 60, 3a)x - 8, b) 5x, c) x - 7, d) 2x + 3
Mini-Lecture 3.3
Solving Linear Equations in One Variable

Learning Objectives:

1. Solve linear equations using the addition and multiplication properties.
2. Solve linear equations containing parentheses.
3. Write numerical sentences as equations.

Examples:

1. Solve each equation. Remember to check your answer by substitution.
   a) $2x - 20 = 0$
   b) $3p + 5 = 4p + 11$
   c) $6y + 21 = 5y + 9$
   d) $10z = 7z + 10 + 2z$
   e) $-2a + 24 = -8a - 6a$
   f) $40 - 5y + 5 = -2y - 10 - 4y$

2. Solve each equation. Remember to check your answer by substitution.
   a) $5(y + 5) = 6(y - 8)$
   b) $3(y + 5) = 4(y - 4)$
   c) $4(2x - 4) = 7(x + 5)$
   d) $6(2a - 3) = 9(a + 4)$
   e) $-7y + 6(-3y - 7) = -64 - 3y$
   f) $6b + 5(-3b - 2) = -12 - 7b$
   g) $-2(8y - 6) - 2(-7y - 3) = -8$
   h) $5(2z - 2) = 9(z + 5)$

3. Write each sentence as an equation.
   a) The sum of $-57$ and $49$ is $8$.
   b) The difference of negative $31$ and $15$ is negative $46$.
   c) The quotient of $-10$ and $2$ amounts to $-5$.

Teaching Notes:

- Encourage students to write out each step rather than doing it in their head.
- Remind students that it does not matter which side you isolate the variable.
- Caution students to take their time using the distributive property.
- Refer students to the textbook for Steps for Solving an Equation.
- Refer students to the textbook for Key Words or Phrases chart.
- Remind students to always check their final answer by substituting it back into the original equation.

Answers: 1a) 10, b) -6, c) -12, d) 10, e) -2, f) -55; 2a) 73, b) 31, c) 51, d) 18, e) 1, f) 1, g) 13, h) 55;
3a) -57 + 49 = -8, b) -31 - 15 = -46, c) -10/2 = -5.
Mini-Lecture 3.4
Linear Equations in One Variable and Problem Solving

Learning Objectives:

1. Write sentences as equations.
2. Use problem-solving steps to solve problems.

Examples:

1. Write each sentence as an equation. Use x to represent "a number". Do not solve.
   a) A number added to -12 equals 15.
   b) Two subtracted from a number amounts to 55.
   c) Ten subtracted from ten times a number is equal to 150.
   d) The product of a number and -4 is twice the sum of the number and 2.
   e) The quotient of 10 and a number is 130.

2. Translate each to an equation and solve the resulting equation.
   a) Six times a number yields 36. Find the number.
   b) A number subtracted from 16 amounts to the quotient of 42 and 6. Find the number.
   c) The difference of -6 times some number and 12 gives -8 times the sum of the number and -8. Find the number.
   d) A Ford Taurus is traveling three times as fast as a Honda CRV. If their combined speed is 96 miles per hour, find the speed of each car.

Teaching Notes:

- Refer students to Key Words and Phrases chart.
- Refer students to Problem-Solving Steps chart.
- Remind students that a phrase is translated into an expression; a sentence is translated into an equation.
- Many students have difficulty translating words into mathematical symbols. This section will be a challenge to most students.

Answers: 1a) x + (-12) = 15, b) x - 2 = 55, c) 10x - 10 = 150, d) -4x=2(x+2), e) 10x=130; 2a) 6x + 36, x = 6, b) 16 - x = 42/6, x = 9, c) -6x - 12 = -8(x + (-8)), x = 38, d) x + 3x = 96; Ford = 72 mph, Honda = 24 mph.
Mini-Lecture 4.1
Introduction to Fractions and Mixed Numbers

Learning Objectives:
1. Identify the numerator and the denominator of a fraction.
2. Write a fraction to represent parts of figures or real-life data.
3. Graph Fractions on a Number Line.
4. Review division properties for 0 and 1.
5. Write mixed numbers as improper fractions.
6. Write improper fractions as mixed numbers or whole numbers.
7. Key Vocabulary: fractions, numerator, denominator, proper fraction, improper fraction, mixed number.

Examples:
1. Identify the numerator and the denominator of a fraction.
   a) \(\frac{3}{7}\)  
   b) \(\frac{12}{13}\)  
   c) \(\frac{10}{7}\)  
   d) \(\frac{13}{13}\)
2. Represent the shaded part and unshaded part of each figure as a fraction.
   a) 
   b) 
   Draw and shade a part of a diagram to represent each fraction.
   c) \(\frac{1}{6}\) of a diagram  
   d) \(\frac{5}{9}\) of a diagram.

Write a fraction to represent the following information.
   c) Of the 207 students taking Basic Mathematics, 143 are freshman. What fraction of the class is freshman?

3. Graph each fraction on a number line.
   a) \(\frac{1}{4}\)  
   b) \(\frac{8}{3}\)  
   c) \(\frac{9}{7}\)  
   d) \(\frac{3}{5}\)

4. Simplify.
   a) \(\frac{4}{4}\)  
   b) \(\frac{-7}{1}\)  
   c) \(\frac{0}{2}\)  
   d) \(\frac{-12}{0}\)

5. Write each mixed number as an improper fraction.
   a) \(3\frac{1}{2}\)  
   b) \(2\frac{8}{9}\)  
   c) \(13\frac{2}{9}\)  
   d) \(103\frac{3}{11}\)

6. Write each improper fraction as a mixed number or a whole number.
   a) \(\frac{16}{3}\)  
   b) \(\frac{38}{5}\)  
   c) \(\frac{156}{12}\)  
   d) \(\frac{559}{143}\)

Teaching Notes:
- Students need to have a firm grasp of fraction vocabulary before continuing.
- Many students confuse \(\frac{0}{x}\) with \(\frac{x}{0}\). Be sure to stress the difference.
- Many students can write a fraction to represent a real-life situation, but they do not truly understand the meaning.

Answers: 1a) n=3, d=7; b) n=12, d=13; c) a=10, d=7; d) n=13, d=13; 2a) 2/5, 3/5; b) 4/8, 4/8; 2c – 2d) diagrams will vary; e) 143/207; 3a) -3d) number lines; 4a) 1; b) -7; c) 0; d) undefined; 5a) 7/2; b) 26/9; c) 119/9; d) 1136/11; 6a) 5/3; b) 73/5; c) 13; d) 116/143
Mini-Lecture 4.2
Factors and Simplest Form

Learning Objectives:

1. Write a number as a product of prime numbers.
2. Write a fraction in simplest form.
3. Determine whether two fractions are equivalent.
4. Solve problems by writing fractions in simplest form.
5. Key Vocabulary: factor, prime factorization, prime numbers, composite number, simplest form, lowest terms.

Examples:

1. Write the prime factorization of each number.
   a) 30  b) 75  c) 170  d) 360

2. Write each fraction in simplest form.
   a) \( \frac{10}{16} \)  b) \( \frac{36}{63} \)  c) \( \frac{77}{88} \)  d) \( \frac{12}{42} \)
   c) \( \frac{11}{34} \)  f) \( \frac{-27}{36} \)  g) \( \frac{30}{80} \)  d) \( \frac{6}{-105} \)

3. Determine whether each pair of fractions is equivalent.
   a) \( \frac{5}{10} \) \ and \ \frac{11}{22} \)  b) \( \frac{7}{21} \) \ and \ \frac{8}{24} \)  c) \( \frac{2}{7} \) \ and \ \frac{8}{15} \)  d) \( \frac{6}{0} \) \ and \ \frac{0}{6} \)

4. Solve. Write each fraction in simplest form.
   a) Alicia was scheduled to work 6 hours at the tanning salon. What fraction of Alicia's shift is represented by 4 hours?
   b) There are 36 inches in a yard. What fraction of a yard is represented by 9 inches?
   c) There are 140 students in a freshman lecture class. If 16 students are absent, what fraction of the students are absent?

Teaching Notes:

- Many students will understand equivalent fractions if they are shown drawings.
- Some students will confuse cross products and simplifying. Stress that cross products is only a check to determine equality of fractions.
- Some students prefer to reduce fractions by factoring the numerator and denominator as products of prime numbers, then canceling all common factors. Others prefer to repeatedly divide the numerator and the denominator by a common factor.

Answers: 1a) 2\( \times \)3\( \times \)5, b) 3\( \times \)5\( \times \)7, c) 2\( \times \)5\( \times \)17, d) 2\( \times \)3\( \times \)5\( \times \)5; 2a) 5/8, b)4/7, c) 7/8, d) -2/7, e) cannot be simplified.
  f) -3/4, g) 3/8, d) -2/35; 3a) yes, b) yes, c) no, d) no; 4a) 2/3; b) 1/4; c) 4/33
Mini-Lecture 4.3
Multiplying and Dividing Fractions

Learning Objectives:
1. Multiply fractions.
2. Evaluate exponential expressions with fractional bases.
3. Divide fractions.
4. Multiply and divide given fractional replacement values.
5. Solve applications that require multiplication of fractions.
6. Key Vocabulary: reciprocal, "of".

Examples:

1. Multiply. Write the product in simplest form.
   a) \( \frac{1}{9} \cdot \frac{1}{7} \)
   b) \( \frac{2}{3} \cdot \frac{1}{4} \)
   c) \( -\frac{5}{6} \cdot -\frac{2}{3} \)
   d) \( \frac{7}{2} \cdot \frac{6}{3} \)
   e) \( \frac{5}{18} \cdot \frac{15}{2} \)
   f) \( \frac{7}{8} \cdot 0 \)
   g) \( \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{1}{5} \)
   h) \( \frac{12}{14} \cdot \frac{3}{9} \cdot \frac{2}{10} \)

2. Evaluate.
   a) \( \left( \frac{1}{2} \right)^2 \)
   b) \( \left( -\frac{1}{3} \right)^4 \)
   c) \( \left( -\frac{2}{5} \right)^3 \)
   d) \( \left( \frac{2}{7} \right)^2 \cdot \frac{1}{4} \)

3. Divide. Write all quotients in simplest form.
   a) \( \frac{3}{5} \div \frac{4}{7} \)
   b) \( \frac{1}{4} \div \frac{1}{4} \)
   c) \( \frac{1}{5} \div \frac{9}{19} \)
   d) \( \frac{8}{17} \div \frac{12}{15} \)
   e) \( \frac{2}{17} \div \frac{3}{17} \)
   f) \( \frac{1}{14} \div 0 \)
   g) \( \frac{27}{4} \div \frac{7}{7} \)
   h) \( 0 \div \frac{3}{11} \)

4. Given the following replacement values, evaluate (a) \( xy \) and (b) \( x + y \).
   a) \( x = -\frac{1}{3} \) and \( y = \frac{4}{9} \)
   b) \( x = \frac{5}{7} \) and \( y = -\frac{5}{9} \)

5. Solve. Write each answer in simplest form.
   a) Find \( \frac{1}{3} \) of 48.
   b) Find \( \frac{3}{7} \) of \(-63\)
   c) A bike trail is 27 miles long. Michelle bikes \( \frac{2}{3} \) of the trail. How many miles did Michelle bike?

Teaching Notes:
- Encourage students to divide out common factors in the numerator and denominator before multiplying.
- When dividing, encourage students to take the time and rewrite the problem by changing the division symbol to multiplication and multiply by the reciprocal. Many students begin "simplifying" and forget to multiply by the reciprocal.

Answers: 1a) 1/63, b) 1/6, c) 5/9, d) -7, e) 3, f) 0, g) -3/50, h) -2/35; 2a) 4/5, b) 1/81, c) -8/125, d) 1/49, 3a) 21/20, b) 1, c) -19/45, d) 10/17, 4) 2/3, 5) undefined, 6) -27/4, h) 0; 4a) -4/27, -3/4; b) -25/63, -9/7; 5a) 16, b) -27, c) 18.
Mini-Lecture 4.4
Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions

Learning Objectives:
1. Add or subtract like fractions.
2. Add or subtract given fractional replacement values.
3. Solve problems by adding or subtracting like fractions.
4. Find the least common denominator of a list of fractions.
5. Write equivalent fractions.
6. Key Vocabulary: like fractions, unlike fractions, (LCD) least common denominator, (LCM) least common multiple, and equivalent fractions.

Examples:
1. Add and simplify.
   a) \( \frac{1}{9} + \frac{4}{9} \)  
   b) \( \frac{1}{8} + \frac{5}{8} \)  
   c) \( \frac{1}{10} + \frac{9}{10} \)  
   d) \( \frac{3}{14} + \left( -\frac{5}{14} \right) \)

   Subtract and simplify.
   e) \( \frac{6}{8} - \frac{3}{8} \)  
   f) \( \frac{6}{21} - \frac{5}{21} \)  
   g) \( \frac{25}{42} - \left( -\frac{7}{42} \right) \)  
   h) \( -\frac{28}{13} - \left( -\frac{5}{13} \right) \)

2. Evaluate each expression if \( x = \frac{3}{5} \) and \( y = -\frac{1}{5} \).
   a) \( x + y \)  
   b) \( x \cdot y \)  
   c) \( x - y \)  
   d) \( x \div y \)

3. Solve.
   a) Find the perimeter of a triangle with sides: \( \frac{6}{25} \) inch, \( \frac{9}{25} \) inch, and \( \frac{5}{25} \) inch.
   b) Corn read \( \frac{2}{11} \) of her book on Friday, \( \frac{3}{11} \) of her book on Saturday, and \( \frac{4}{11} \) of her book on Sunday. What part of her book has she read?

4. Find the LCM of each list of numbers.
   a) 10, 12  
   b) 12, 14  
   c) 30, 35  
   d) 30, 20, 50

5. Write each fraction as an equivalent fraction with the given denominator.
   a) \( \frac{4}{9} = \frac{-}{18} \)  
   b) \( \frac{7}{11} = \frac{-}{55} \)  
   c) \( \frac{5}{2} = \frac{-}{6} \)  
   d) \( \frac{2}{3} = \frac{-}{24} \)

Teaching Notes:
- Many students add or subtract both numerator and denominator.
- When subtracting, some students may need to take the intermediate step of writing out the operations performed on the numerators. For example: \( -\frac{1}{7} - \left( -\frac{3}{7} \right) = -\frac{1}{7} + \left( -\frac{3}{7} \right) = \frac{-1 + 3}{7} = \frac{2}{7} \).
- Some students forget to multiply the numerator when building equivalent fractions.

Answers: a) 5, 9, b) 7, c) 4, 5, d) -4, 7, e) 3, 8, f) -11, 21, g) 16, 21, h) -23, 13, 2a) 2, 5, b) -3, 25, c) 4, 5, d) -3, 2a) 4, 5, inch, b) 9, 11, 4a) 60, b) 34, c) 210, d) 300, 5a) 8, b) 35, c) 15, d) 16.
Mini-Lecture 4.5
Adding and Subtracting Unlike Fractions

Learning Objectives:
1. Add or subtract unlike fractions.
2. Write fractions in order.
3. Evaluate expressions given fractional replacement values.
4. Solve problems by adding or subtracting unlike fractions.
5. Key Vocabulary: least common denominator (LCD).

Examples:
1. Add or subtract as indicated.
   a) $\frac{1}{10} + \frac{2}{5}$  
   b) $\frac{1}{5} + \left(\frac{2}{25}\right)$  
   c) $\frac{1}{7} + \left(-\frac{9}{10}\right)$  
   d) $\frac{3}{5} + \frac{1}{20}$  
   e) $\frac{4}{5} - \frac{3}{20}$  
   f) $\frac{7}{9} - \left(-\frac{1}{12}\right)$  
   g) $\frac{-5}{7} - \left(-\frac{1}{2}\right)$  
   h) $\frac{1}{20} - \frac{8}{15}$  
   i) $\frac{7}{5} + \frac{8}{16} - \frac{4}{20}$

2. Insert < or > to form a true sentence.
   a) $\frac{2}{3} \frac{1}{9}$  
   b) $\frac{5}{12} \frac{1}{2}$  
   c) $\frac{5}{6} \frac{4}{5}$

3. Evaluate each expression if $x = \frac{1}{4}$ and $y = -\frac{3}{5}$.
   a) $x + y$  
   b) $x \cdot y$  
   c) $x - y$  
   d) $x + y$

4. Solve.
   a) Find the perimeter of a rectangle with width $\frac{3}{4}$ feet and length $\frac{3}{14}$ feet.
   b) Sharon is making matching holiday outfits for her three children. Each outfit required $\frac{7}{8}$ yards. How many yards of material will be needed to make the three outfits?

Teaching Notes:
- Refer students back to Section 3.4: Method 1: Finding the LCM of a List of numbers Using Multiples of the Largest Number and Method 2: Finding the LCM of a List of Numbers Using Prime Factorization.
- Some students try to cross-cancel when adding or subtracting.
- Some students add and subtract both the numerator and denominator.
- Some students forget to multiply the numerator when building equivalent fractions.

Answers: 1a) $\frac{3}{2}$, b) $-\frac{7}{25}$, c) $-\frac{53}{70}$, d) $\frac{13}{20}$, e) $\frac{13}{20}$, f) $\frac{31}{36}$, g) $-\frac{3}{14}$, h) $-\frac{29}{60}$, i) $-\frac{7}{10}$; 2a) $>$, b) $<$, c) $<$; 3a) $-\frac{7}{20}$, b) $-\frac{3}{20}$, c) $\frac{17}{20}$, d) $-\frac{3}{12}$; 4a) 27/14 feet, b) 21/8 yd
Mini-Lecture 4.6
Complex Fractions and Review of Order of Operations

Learning Objectives:
1. Simplify complex fractions.
2. Review the order of operations.
3. Evaluate expressions given replacement values.

Examples:

1. Simplify each complex fraction.
   a) \( \frac{\frac{1}{6}}{\frac{2}{3}} \)  
   b) \( \frac{\frac{16}{7}}{\frac{8}{7}} \)  
   c) \( \frac{\frac{1}{6} + \frac{2}{3}}{\frac{1}{3} + \frac{1}{4}} \)

2. Use the order of operations to simplify each expression.
   a) \( \frac{1}{4} + \frac{1}{4} - \frac{1}{3} \)  
   b) \( 3 \div \left( \frac{7}{8} + \frac{7}{16} \right) \)  
   c) \( \frac{2}{7} + \frac{3}{14} \left( \frac{2}{7} - \frac{3}{14} \right) \)
   d) \( \left( \frac{2}{5} \right)^2 \cdot \frac{1}{2} \)  
   e) \( \frac{1}{2} + \left( \frac{2}{3} \right)^2 - \frac{1}{3} \)  
   f) \( \frac{2}{3} \cdot \left( \frac{1}{4} + \frac{1}{2} \right) \cdot 6 \)

3. Evaluate each expression if \( x = -\frac{1}{2}, \quad y = \frac{3}{5}, \) and \( z = \frac{7}{10}. \)
   a) \( 3x - z \)  
   b) \( \frac{y}{z} \)  
   c) \( x^2 + 2y \)  
   d) \( \frac{x + y}{z} \)

- Many students make careless errors when using Method 2 for simplifying complex fractions. If this is the case, encourage students to use Method 1 (rewrite as a division problem).
- Remind students that when dividing fractions, you must change division to multiplication and multiply by the reciprocal.
- Some students will try to apply procedures for simplifying complex fractions to adding and subtracting fractions.

Answers:  
1a) \( \frac{1}{6} \), b) 2, c) \( \frac{8}{13} \);  
2a) \( \frac{1}{3} \), b) \( \frac{8}{7} \), c) \( \frac{1}{28} \), d) \( \frac{2}{25} \);  
3a) \( -\frac{11}{5} \), b) 6/7, c) 29/20, d) 7/7
Mini-Lecture 4.7
Operations On Mixed Numbers.

Learning Objectives:

1. Graph positive and negative fractions and mixed numbers.
2. Multiply or divide mixed or whole numbers.
3. Add or subtract mixed numbers.
4. Solve problems containing mixed numbers.
5. Perform operations on negative mixed numbers.

Examples:

1. Graph each list of numbers on a number line.
   a) \(-3, -3 \frac{1}{2}, -1, \frac{3}{4}, 2\)
   b) \(4, -3 \frac{3}{4}, 0, 1 \frac{1}{5}, - \frac{1}{2}\)

2. Multiply or divide.
   a) \(\frac{2}{3} \times \frac{1}{2}\)
   b) \(\frac{3}{4} \div \frac{3}{5}\)
   c) \(4 \frac{1}{5} \div \frac{1}{5}\)
   d) \(3 \frac{1}{3} + 2 \frac{3}{5}\)

3. Add or subtract.
   a) \(10 \frac{1}{2} + 7 \frac{1}{9}\)
   b) \(6 \frac{1}{3} + 13 \frac{4}{9}\)
   c) \(4 \frac{2}{3} + 9 \frac{7}{9}\)
   d) \(8 \frac{1}{3} + 2 \frac{2}{3} + 3 \frac{2}{9}\)
   e) \(15 \frac{8}{9} - 6 \frac{2}{9}\)
   f) \(19 \frac{1}{25} - 7 \frac{1}{5}\)
   g) \(17 \frac{1}{6} - 5 \frac{13}{24}\)
   h) \(13 - 6 \frac{5}{9}\)

4. Solve. Write answer in simplest form.
   a) Amy rode her bicycle \(\frac{9}{15}\) miles on each of 9 days. What is the total distance Amy rode?
   b) John cuts a board \(13 \frac{3}{7}\) feet long from one 20 feet long. How long is the remaining piece?

5. Perform the indicated operation.
   a) \(-5 \frac{7}{8} \div 5 \frac{1}{4}\)
   b) \(-7 \frac{7}{9} + \left(-4 \frac{2}{9}\right)\)
   c) \(-8 \frac{1}{3} \times \left(-1 \frac{1}{5}\right)\)
   d) \(5 \frac{5}{8} \div (-9)\)
   e) \(19 \frac{3}{5} - \left(-5 \frac{18}{20}\right)\)
   f) \(-15 \frac{3}{7} + 14 \frac{1}{5}\)
   g) \(10 \frac{1}{9} + \left(-5 \frac{5}{9}\right)\)
   h) \(-9 \times \left(5 \frac{7}{12}\right)\)

Teaching Notes:

- Most students forget that mixed numbers must be changed to improper fractions before multiplying. Some try to multiply the whole number parts together, and then multiply the fractional parts together.
- Many students confuse the rules for multiplication with adding/subtracting rules.
- Many students are challenged by word problems. Students have trouble deciding which operation to use for the word problems.

Answers: 1a) \(-\frac{1}{4}\); 1b) \(-\frac{2}{3} \div -2 \frac{1}{2}\); 2a) \(4 \div 3\); 6; c) 21; d) \(1 \frac{1}{3}\);

3a) 17; 11/18; b) \(19 \div 7\); c) \(14 \div 49\); d) \(14 \div 29\); e) \(9 \div 23\); f) \(11 \div 21\); g) 115/8; h) 6/49; 4a) 83 2/5; b) \(6 \div 47\); 5a) \(-47 \div 42\); b) \(-12\); c) 10; d) \(-5 \div 8\); e) \(25 \div 2\); f) \(-1 \div 8\); g) \(4 \div 5\); h) \(-10 \div 5\).
Mini-Lecture 4.8
Solving Equations Containing Fractions

Learning Objectives:

1. Solve Equations Containing Fractions.
2. Solve Equations by Multiplying by the LCD.
3. Review Adding and Subtracting Fractions.

Examples:

1. Solve each equation. Check your proposed solution.
   a. \( \frac{1}{2} \cdot x = \frac{1}{2} \)
   b. \( \frac{4}{15} - \frac{3}{15} = \frac{1}{5} \)
   c. \( -\frac{3}{8} = x - \frac{5}{6} \)
   d. \( 8x - \frac{3}{5} - 7x = \frac{11}{15} \)

2. Solve each equation.
   a. \( \frac{1}{3} x = 5 \)
   b. \( -\frac{3}{8} x = -\frac{4}{5} \)
   c. \( -8a = \frac{16}{25} \)
   d. \( \frac{3}{5} y = -\frac{7}{25} \)
   e. \( \frac{x}{7} - x = -5 \)
   f. \( \frac{b}{3} = \frac{b + 7}{5} \)

3. Add or subtract as indicated.
   a. \( \frac{n + 4}{2} \)
   b. \( \frac{5c - c}{8} \)

Teaching Notes:

- Emphasis checking proposed solutions.
- When adding or subtracting fractions, the denominators need to be the same.
- Review properties: Addition Property of Equality
  Multiplication Property of Equality
- Make sure students understand the difference between solving an equation containing fractions
  (multiply both sides of the equation by the LCD of the fractions) and adding or subtracting two fractions (create equivalent fractions).

Answers: 1a) -1,  b) \( \frac{7}{15} \),  c) \( \frac{11}{24} \),  d) \( \frac{4}{3} \);  2a) 15,  b) \( \frac{32}{15} \),  c) \( -\frac{2}{25} \),  d) \( -\frac{7}{15} \),  e) \( \frac{35}{6} \),  f) \( \frac{35}{2} \);  3a) \( \frac{7n + 8}{14} \),  b) \( \frac{3c}{8} \)