

Mini-Lecture 5.1

Decimal Applications: Introduction to Decimals

Learning Objectives:

1. Know the meaning of place value for a decimal number, and write decimals in words.
2. Write decimals in standard form.
3. Write decimals as fractions.
4. Compare decimals.
5. Round decimals to a given place value.
6. Key Vocabulary: *decimals, standard form, < (less than), > (greater than), = (equal to), round the decimal part.*

Examples:

1. Determine the place value for the digit 9 in each number.
a) 90 b) 900 c) 0.9 d) 0.09

Write each decimal number in words.
e) 8.54 f) -0.382 g) 7002.09
2. Write each decimal number in standard form.
a) two and seven tenths b) negative eleven and five hundredths
c) seven hundred three and two hundred fifty-five thousandths d) negative ninety-five ten thousandths
3. Write each decimal as a fraction or a mixed number. Write your answer in simplest form.
a) 0.7 b) -0.35 c) 0.094 d) -2.4005
4. Insert $<$, $>$, or $=$ to form a true statement.
a) 0.2 _____ 0.5 b) 0.14 _____ -0.14000
c) 0.6401 _____ 0.6410 d) -15.0037 _____ 15.00037
5. Round each decimal to the given place value.
a) 0.39 to the nearest tenth b) -0.174 to the nearest hundredth
c) 1.4782 to the nearest thousandth d) -22.099 to the nearest hundredth

Round each monetary amount to the nearest cent or dollar as indicated.
e) \$0.058 to the nearest cent f) \$17.88 to the nearest dollar

Teaching Notes:

- Most students find problems 1 and 2 easy.
- Some students have difficulty with example 3 when a whole number is involved.
- Some students become confused when rounding monetary values. When rounding to the nearest cent, it is important to remind them that this is the hundredths position (one-hundredths-of-a-dollar position).

Answers: 1a) tens, b) hundreds, c) tenths, d) hundredths, e) eight and fifty-four hundredths, f) negative three hundred eighty-two thousandths, g) seven thousand two and nine hundredths; 2a) 2.7, b) -11.05, c) 703.255, d) -0.0095;

3a) $\frac{7}{10}$, b) $-\frac{7}{20}$, c) $\frac{47}{500}$, d) $-2\frac{801}{2000}$; 4a) $<$, b) $>$, c) $<$, d) $<$; 5a) 0.4, b) -0.17, c) 1.478, d) -22.10, e) \$0.06, f) \$18

Mini-Lecture 5.2

Adding and Subtracting Decimals

Learning Objectives:

1. Add or subtract decimals.
2. Estimate when adding or subtracting decimals.
3. Evaluate expressions with decimal replacement values.
4. Simplify expressions containing decimals.
5. Solving problems that involve adding and subtracting decimals.

Examples:

1. Add. Be sure to estimate to see if the answer is reasonable.

a) $0.5 + 0.1$

b) $-2.7 + -3.2$

c) $7.2 + 3.27$

d) $-372 + 9.302$

e) $43.097 + 289.3887$

f) $5.03 + 16.988 + 0.006$

2. Subtract. Be sure to estimate to see if the answer is reasonable.

a) $0.8 - 0.2$

b) $-7.5 - 2.3$

c) $187.5 - 8.39$

d) $8.2 - 5.006$

e) $-632.021 - (-295.9)$

f) $1000 - 3.0947$

3. Evaluate each expression for $x = 2.4$, $y = 3$, and $z = 0.51$.

a) $x + z$

b) $y - x$

c) $x + y - z$

4. Solve.

a) Recently, Allison went shopping and spent \$18.92 at the bookstore, \$68.03 at the grocery store, and \$129.76 at a department store. What is the total amount of money Allison spent?

b) Find the perimeter of a rectangular lawn that measures 40.93 feet by 27.09 feet.

Teaching Notes:

- Remind students to work in a vertical format and line-up the decimal point and corresponding place values.
- Some students need to be shown how to add extra zeros to the ends of the decimal part of the numbers and where to place the decimal point with whole numbers.
- Some students must be reminded of how to borrow across zeros when subtracting.
- Refer students to the **Adding or Subtracting** decimals chart in the textbook.

Answers: 1a) 0.6, b) -5.9, c) 10.47, d) -362.698, e) 332.4857, f) 22.024; 2a) 0.6, b) -9.8, c) 179.11, d) 3.194, e) -336.121, f) 996.9053; 3a) 2.91, b) 0.6, c) 4.89; 4a) \$216.71, b) 136.04 ft

Mini-Lecture 5.3

Multiplying Decimals and Circumference of a Circle

Learning Objectives:

1. Multiply decimals.
2. Estimate when multiplying decimals.
3. Multiply by powers of 10.
4. Evaluate expressions with decimal replacement values.
5. Find the circumference of a circle. ($\pi \approx 3.14$ or $\frac{22}{7}$)
6. Solve problems by multiplying decimals.
7. Key Vocabulary: π (*pi*), *perimeter*, *circumference*, *diameter*, *radius*.

Examples:

1. Multiply.

a) 0.7×0.2

b) 1.33×-0.5

c) 7.2×5.8

d)
$$\begin{array}{r} 0.856 \\ \times 3.1 \\ \hline \end{array}$$

e)
$$\begin{array}{r} -2.00033 \\ \times -6.9 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 0.0896 \\ \times 0.345 \\ \hline \end{array}$$

2. Multiply. Check by estimating.

a) $(6.8)(3.2)$

b) $(8.4)(1.8)$

c) $(5.8)(0.7)$

3. Multiply.

a) 4.3×10

b) 17.693×100

c) -0.0027×1000

d) -0.07×-0.1

e) 9.07×0.01

f) 2.908×0.001

4. Evaluate each expressions for $x = 2$, $y = -0.3$, and $z = 7.3$.

a) xy

b) $xz - y$

c) $-3y + z$

5. Find the circumference of a circle with the given information. Use $\pi = 3.14$.

a) radius = 7 feet

b) diameter = 16 inches

c) radius = 10.3 meters

6. a) Write 57.6 million in standard form.

b) A 1-ounce serving of hot cocoa contains 0.375 grams of fat. How many grams of fat are in an 8 oz. mug of hot cocoa?

Teaching Notes:

- Some students do not see the pattern that develops when multiplying by powers of 10, they must be shown.
- Many students prefer to multiply numbers by a power of ten the long way.
- Some students will attempt to line up the decimal point (like adding) when multiplying.
- Refer students to $C = 2\pi r$.

Answers: 1a) 0.14, b) -0.665, c) 41.76, d) 2.6536, e) 13.802277, f) 0.030912; 2a) 21.76; b) 15.12; c) 4.06; 3a) 43, b) 1769.3, c) -2.7, d) 0.007, e) 0.0907, f) 0.002908; 4a) -0.6, b) -6.7, c) 8.2; 5a) 43.96 ft. b) 50.24 in., c) 64.684 m; 6a) 57,600,000; b) 3g

Mini-Lecture 5.4

Dividing Decimals

Learning Objectives:

1. Divide decimals.
2. Estimate when dividing decimals.
3. Divide decimals by powers of 10.
4. Evaluate expressions with decimal replacement values.
5. Solve problems by dividing decimals.

Examples:

1. Divide.

a) $1.5 \div 5$

b) $26 \overline{)7.826}$

c) $-518 \overline{)0.9324}$

d) $8.9 \overline{)22.25}$

e) $-1411.51 \div -36.1$

f) $0.02 \overline{)0.8}$

2. Divide. Then estimate to see if your answer is reasonable.

a) $2.5 \overline{)18.5}$

b) $2.4 \overline{)35.4}$

3. Divide decimals by powers of 10.

a) $\frac{7.74}{10}$

b) $1000 \overline{)-887.73}$

c) $1.047 \div 100$

4. Evaluate each expression for $x = 3.02$, $y = -0.3$, and $z = 1.51$.

a) $z \div y$

b) $z \div x$

c) $x \div y$

5. Solve.

- a) Divide 0.894 by -0.041 and round the quotient to the nearest hundredth.

- b) Preparing for a picnic, Carol went to the Deli and purchased: 0.52 pounds of salami at \$3.29/pound; 0.48 pounds of sliced turkey at \$8.99/pound; 1.04 pounds of ham at \$3.99/pound; and 0.98 pounds of cheese at \$4.29/pound. What was the total amount Carol spent at the deli? (Round your answer to the nearest cent.)

Teaching Notes:

- Most students have forgotten the mechanics of long division with decimals.
- Stress the importance of neatness so that the decimal ends up in the correct position. Some students find it helpful to do the division on graph paper or on lined paper that is rotated so the lines are vertical.
- Remind students when rounding the quotient to a specific place, you need to carry your division one more place than the rounding place.

Answers: 1a) 0.3, b) 0.301, c) -0.0018, d) 2.5, e) 39.1, f) 40; 2a) 7.4; b) 14.75; 3a) 0.774, b) -0.88773, c) 0.01047; 4a) -5.03, b) 0.5, c) -10.06; 5a) -21.81, b) \$14.38

Mini-Lecture 5.5

Fractions, Decimals, and Order of Operations

Learning Objectives:

1. Write fractions as decimals.
2. Compare fractions and decimals.
3. Simplify expressions containing decimals and fractions using order of operations.
4. Solve area problems containing fractions and decimals.
5. Evaluate expressions given decimal replacement values.

Examples:

1. Write each fraction or mixed number as a decimal.

a) $\frac{3}{5}$

b) $-\frac{3}{20}$

c) $\frac{1}{3}$

d) $\frac{5}{16}$

e) $-\frac{13}{11}$

f) $-1\frac{7}{8}$

2. Insert $<$, $>$, or $=$ to form a true statement.

a) -0.0832 _____ -0.0823

b) 0.501 _____ $\frac{1}{2}$

c) 0.428 _____ $\frac{3}{7}$

Write the numbers in order from smallest to largest.

d) 0.331 , $\frac{1}{3}$, 0.330

e) 2.15 , 2.142 , $\frac{15}{7}$

f) 1.5833 , $1\frac{21}{36}$, $\frac{38}{36}$

3. Simplify each expression.

a) $(0.3)^2 - 0.4$

b) $(7.3)(100) - (7.2)(10)$

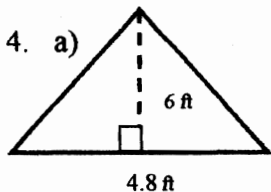
c) $\frac{4 + 0.42}{-2}$

d) $\frac{1}{4} - 3(6.5)$

e) $\frac{3}{5} - (6.4)(-3)$

f) $\frac{1}{5}(-9.1 - 3.3)$

4. a)



5. Evaluate for $x = -2$, $y = 0.5$, and $z = 3.6$:

a) $\frac{x}{y} + 2z$

Teaching Notes:

- Most students need a review of order of operations.
- Most students, once taught how to convert from fraction to decimal, have little problems.
- Some students have difficulty ordering numbers when they are mixed. Suggest that after converting to decimal, line the decimal points up vertically and compare corresponding place value.

Answers: 1a) 0.6, b) -0.15, c) $0.\bar{3}$, d) 0.3125, e) $-1.\bar{18}$, f) -1.875; 2a) $<$, b) $>$ c) $<$, d) 0.330, 0.331, $\frac{1}{3}$, e) 2.142, $\frac{15}{7}$, 2.15, f) $\frac{38}{36}$, 1.5833, $1\frac{21}{36}$; 3a) -0.31, b) 658, c) -2.21, d) -19.25, e) 19.8, f) -2.48; 4a) 14.4ft; 5a) 3.2

Mini-Lecture 5.6

Equations Containing Decimals

Learning Objectives:

1. Solve Equations Containing Decimals

Examples:

1. Solve each equation.

a. $z + 0.8 = 2.5$

b. $0.27x = -0.81$

c. $3.9 = 1.5 + 0.6x$

d. $5x + 3.6 = 8x + 12.9$

e. $6.8 - 4x = 5(x - 13.7)$

Teaching Notes:

- Review steps for solving equations involving a variable.
- Stress neatness of lining up decimals when adding or subtracting, especially when an integer is present.
- Remind students the advantage of multiplying by powers of 10 to eliminate decimals when solving equations.
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Answers: 1a) 1.7, b) -0.3, c) 4, d) -3.1, e) 3.7

Mini-Lecture 5.7

Decimal Applications: Mean, Median, and Mode

Learning Objectives:

1. Find the mean of a list of numbers.
2. Find the median of a list of numbers.
3. Find the mode of a list of numbers.
4. Key Vocabulary: *measures of central tendency, mean, median, mode, grade point average (GPA), weighted mean.*

Examples:

1. Find the mean of the list of numbers. If necessary, round to the nearest tenth.

a) 71, 47, 71, 99, 47 b) 143, 83, 225, 16 c) 4.1, 1.4, 8.7, 1.9, 13.1, 7.6

d) Find the GPA (grade point average) for a particular student. Use the following for point values of the grades: A-4, B-3, C-2, D-1, F-0. If necessary, round the grade point average to the nearest hundredth.

Grade	Credit Hours
A	3
B	3
A	4
C	3

2. Find the median of the following list of numbers. Round to the nearest hundredth, if necessary.

a) 3, 3, 14, 27, 31, 37, 50 b) 10, 2, 1, 28, 45, 48, 36
c) 8, 2, 21, 19, 23, 49, 39, 38 d) 0.1, 3.3, 2.5, 0.3, 4, 2.7

3. Find the mode of the following list of numbers.

a) 20, 43, 46, 43, 49, 43, 50 b) 90, 57, 32, 57, 29, 90
c) 5, 9, 46, 3, 2, 8, 18, 1, 6, 19 d) 7.08, 7.41, 7.56, 7.08, 7.88, 7.99, 7.62

Teaching Notes:

- Most students are familiar with mean (average) but have not worked with median or mode.
- Encourage students to estimate when finding the mean.
- Stress the importance of ordering the information when finding median and mode.

Answers: 1a) 67, b) 116.8, c) 6.1, d) 3.31; 2a) 27, b) 28, c) 22, d) 2.6; 3a) 43, b) 57,90, c) no mode, d) 7.08

Mini-Lecture 6.1

Ratio and Rates

Learning Objectives:

1. Write ratios as fractions.
2. Write rates as fractions.
3. Find unit rates.
4. Find unit prices.
5. Key Vocabulary: *ratio, rates, unit rate, miles per hour, unit price.*

Examples:

1. Write each ratio as a fraction in simplest form. Write the fraction in simplest form.

- | | | |
|--------------------------|------------------------------------|-------------------------------------|
| a) 20 to 30 | b) 16 to 10 | c) 1.5 to 10 |
| d) 1.17 days to 9.9 days | e) $3\frac{1}{4}$ hours to 6 hours | f) $7\frac{1}{5}$ to $7\frac{1}{3}$ |

2. Write each rate as a fraction in simplest form.

- | | |
|----------------------------|----------------------------|
| a) 81 miles in 42 minutes | b) 1036 miles in 63 hours |
| c) 6 tests for 30 students | d) 246 miles on 54 gallons |

3. Write each rate as a unit rate.

- | | |
|--------------------------------|-----------------------------------|
| a) 84 miles in 4 hours | b) 468 miles on 18 gallons of gas |
| c) 1161 cars in 387 households | d) 380 people in 10 buses |

4. Find each unit price. Round to the nearest cent, if necessary.

- | | |
|-----------------------------------|--------------------------|
| a) \$0.90 for 10 ounces | b) \$1.39 for 17 ounces |
| c) \$11.92 for 8 pounds of apples | d) \$118.58 for 121 pens |
- e) Which is the better buy (lowest cost per ounce)? Round to the thousandth, if necessary. Shampoo: \$0.70 for 11 ounces or 17 ounces for \$1.39.
- f) There were 3 men and 6 women on a volleyball team. Find the ratio of men to women.

Teaching Notes:

- Some students need a quick review on converting mixed numbers to improper fractions and dividing fractions, decimals, etc.
- Remind students that order is important when solving ratio and rate word problems.
- Remind students that unit rate has a denominator of one.

Answers: 1a) $\frac{2}{3}$, b) $\frac{8}{5}$, c) $\frac{3}{20}$, d) $\frac{13}{110}$, e) $\frac{13}{24}$, f) $\frac{54}{55}$; 2a) 27miles/14minutes, b) 148 miles/9hours, c) 1 test for 5 students, d) 41 miles per 9 gallons; 3a) 21 miles/hr, b) 26 miles/gallons, c) 3 cars/household, d) 38 people/bus; 4a) \$0.09/oz, b) \$0.08/oz, c) \$1.49/pound, d) \$0.98/pen; e) 11 ounces for \$0.07, f) $\frac{1}{2}$

Mini-Lecture 6.2

Proportions

Learning Objectives:

1. Write sentences as proportions.
2. Determine whether proportions are true.
3. Finding the unknown number in a proportion.
4. Key Vocabulary: *proportion, cross products.*

Examples:

1. Write each sentence as a proportion.

a) 70 pencils is to 28 students as 5 pencils is to 2 students

b) 96 guests is to 12 tables as 8 guests is to 1 student.

c) $2\frac{1}{11}$ ounces of pasta is to 3 grams of protein as $12\frac{6}{11}$ ounces of pasta is to 18 grams of protein.

2. Determine whether each proportion is a true proportion.

a) $\frac{3}{6} = \frac{4}{8}$

b) $\frac{1\frac{10}{13}}{3} = \frac{3\frac{7}{13}}{6}$

c) $\frac{0.6}{0.5} = \frac{3.0}{2.6}$

d) $\frac{2\frac{2}{9}}{7} = \frac{8\frac{8}{9}}{30}$

3. For each proportion, find the unknown number n.

a) $\frac{n}{54} = \frac{1}{18}$

b) $\frac{1}{-2} = \frac{n}{15}$

c) $\frac{1}{9\frac{1}{2}} = \frac{n}{19}$

d) $\frac{-4.5}{n} = \frac{-2.5}{5.5}$

Teaching Notes:

- It is important to stress the easy problems 1a, 1b, and 2a to show that they are equivalent fractions.
- Many students have difficulty when complex fractions are involved in the proportion.
- Many students need to actually circle the cross products so they can be clear of the procedure.

Answers: 1a) $\frac{70}{28} = \frac{5}{2}$, b) $\frac{96}{12} = \frac{8}{1}$, c) $\frac{2\frac{1}{11}}{3} = \frac{12\frac{6}{11}}{18}$; 2a) yes, b) yes, c) no, d) no; 3a) 3, b) -7.5 or $7\frac{1}{2}$, c) 2, d) 9.9

Mini-Lecture 6.3

Proportions and Problem Solving

Learning Objectives:

1. Solve problems by writing proportions.

Examples:

1. On a map of the city of Worcester, 1-inch corresponds to 12 miles. How far away is a town if the distance on the map measures 3 inches?
2. The ratio of students to faculty is 25 to 2. How many faculty members will be needed for a student population of 623? Round to the nearest whole number.
3. A particular piece of machinery made 1200 revolutions in 20 minutes. How many revolutions will occur in 23 minutes?
4. Four cups of water is needed to make 9.2 cups of rice. How many cups of rice can be made with 21 cups of water?
5. If 2 servings of a recipe calls for $\frac{3}{4}$ teaspoon of butter, how many servings can be made from 9 teaspoons of butter?
6. One bag of lawn & garden fertilizer covers 1000 square feet of lawn. How many bags of fertilizer must you purchase to cover a lawn 440 feet by 220 feet? Remember, you cannot purchase a fractional part of a bag.

Teaching Notes:

- Many students have trouble correctly setting up the proportion. Students tend to write the numbers as they appear rather than “lining up” the units.
- Remind students to estimate to see if their answer is reasonable.
- Refer students to the four steps for problem solving: Understand, Translate, Solve, and Interpret.

Answers: 1) 36 miles, 2) 50 faculty members; 3) 1380 revolutions; 4) 48.3 cups; 5) 24 servings; 6) 97 bags.

Mini-Lecture 6.4

Square Roots and the Pythagorean Theorem

Learning Objectives:

1. Find the square root of a number.
2. Approximate square roots.
3. Use the Pythagorean Theorem.
4. Key Vocabulary: *square root, radical sign, positive square root, Pythagorean Theorem, leg, hypotenuse.*

Examples:

1. Find each square root.

a) $\sqrt{4}$

b) $\sqrt{25}$

c) $\sqrt{49}$

d) $\sqrt{81}$

e) $\sqrt{\frac{1}{64}}$

f) $\sqrt{\frac{16}{25}}$

g) $\sqrt{\frac{25}{49}}$

2. Use a calculator or the appropriate Appendix to approximate each square root to the thousandths position.

a) $\sqrt{2}$

b) $\sqrt{6}$

c) $\sqrt{11}$

d) $\sqrt{39}$

3. Sketch each right triangle and find the length of the side not given. If necessary, round the answer to the nearest thousandth.

a) leg = 3, leg = 4 b) leg = 12, hypotenuse = 15 c) hypotenuse = 6.4, leg = 3

Solve. If necessary, round to the nearest thousandth.

- d) A section of land is a square with each side measuring 2 miles. Find the length of the diagonal of the section of land.

- e) A garden is in the shape of a rectangle. The diagonal length of the garden is 25 feet, and the length of one of the sides is 15 feet. Find the length of the other side.

Teaching Notes:

- Some students have never done square roots on a calculator and will need guidance.
- When approximating square roots, encourage students to mentally estimate the answer. That way if they use the calculator incorrectly they might be able to notice the incorrect result.
- Most students do not have trouble using the Pythagorean Theorem for finding a hypotenuse, but some have trouble using it for finding a missing leg.

Answers: 1a) 2, b) 5, c) 7, d) 9, e) 1/8, f) 4/5, g) 5/7; 2a) 1.414, b) 2.449, c) 3.317, d) 6.245; 3a) 5, b) 9, c) 5.653; d) 2.828 mi, e) 20 ft

Mini-Lecture 6.5

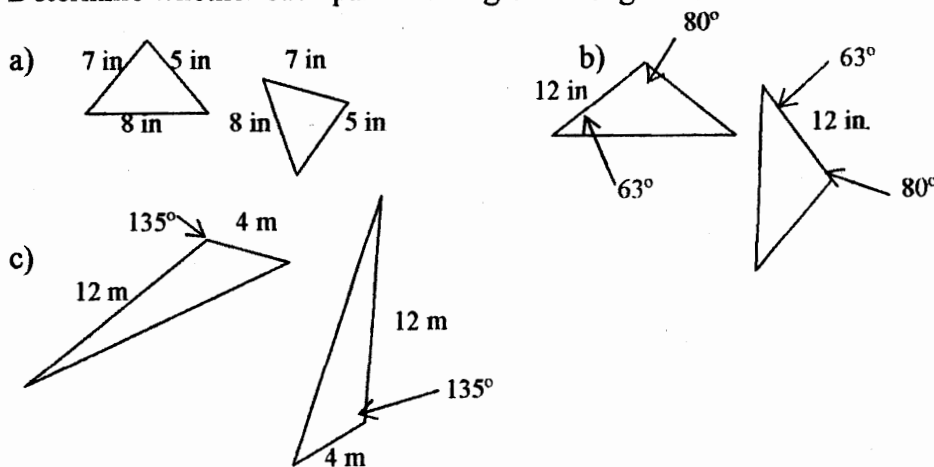
Congruent and Similar Triangles

Learning Objectives:

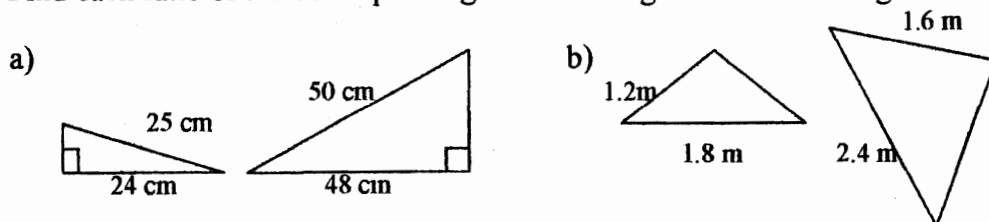
1. Determine whether two triangles are congruent.
2. Find the ratio of corresponding sides in similar triangles.
3. Find unknown length of sides in similar triangles.
4. Key Vocabulary: *congruent*, *Angle-Side-Angle (ASA)*, *Side-Side-Side (SSS)*, *Side-Angle-Side (SAS)*, *similar*.

Examples:

1. Determine whether each pair of triangles is congruent.



2. Find each ratio of the corresponding sides of the given similar triangles.



3. Solve. Round to the nearest tenth.

- a) A tree casts a shadow of 26 ft. Nearby, a 5-ft pole casts a shadow of 3 ft. What is the height of the tree?
- b) A rock climber is 6 feet tall and his shadow measures 9 feet long. The rock she wants to climb casts a shadow of 580 feet. How tall is the rock?

Teaching Notes:

- Some students need a review on solving proportions.
- Encourage students to mentally visualize applied problems and draw and label a diagram before solving.

Answers: 1a) yes, b) yes, c) yes; 2a) $\frac{1}{2}$, b) $\frac{4}{3}$; 3a) 43 $\frac{1}{3}$ ft., b) 386 $\frac{2}{3}$ ft