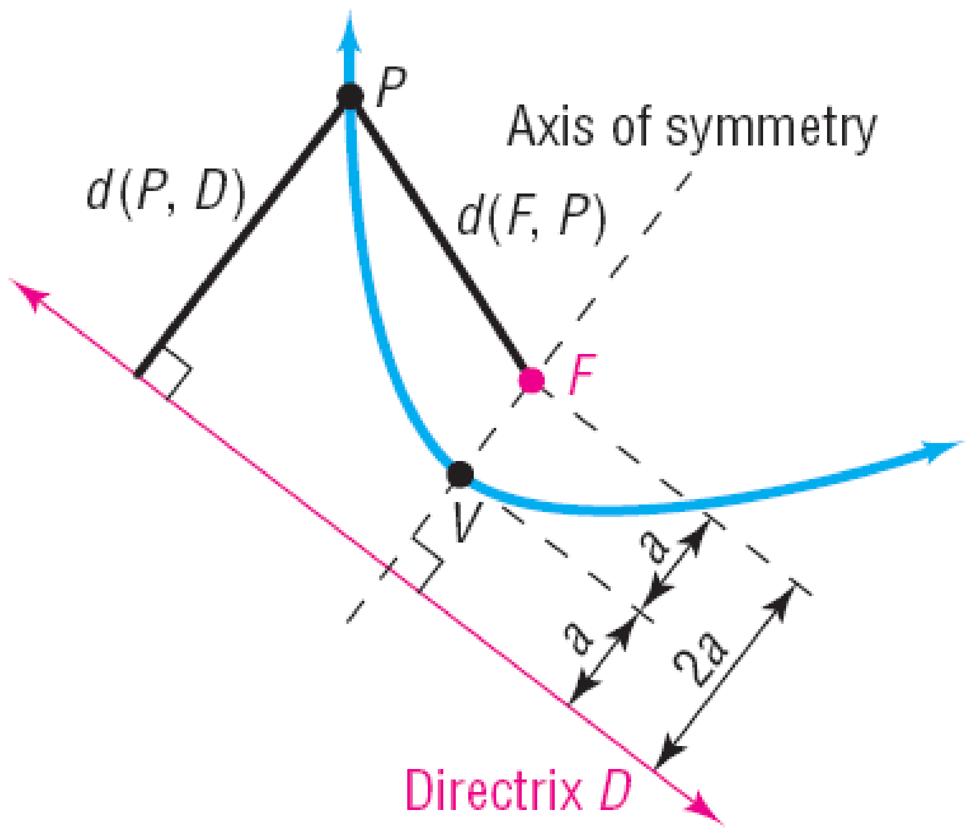
Section 11.2

The Parabola

A **parabola** is the collection of all points P in the plane that are the same distance from a fixed point F as they are from a fixed line D. The point F is called the **focus** of the parabola, and the line D is its **directrix**. As a result, a parabola is the set of points P for which

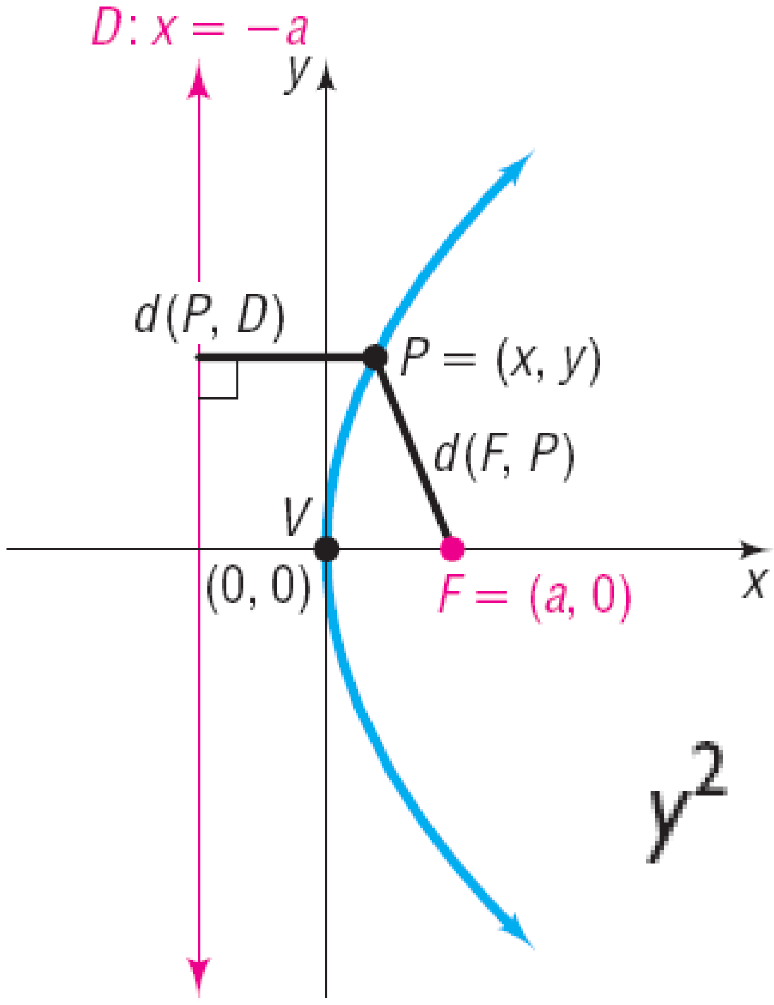
$$d(F, P) = d(P, D) \qquad \textbf{(1)}$$



OBJECTIVE 1

1 Analyze Parabolas with Vertex at the Origin

$$d(F, P) = d(P, D)$$



$$y^2 = 4ax$$

Theorem

Equation of a Parabola

Vertex at (0, 0), Focus at (a, 0), $a > 0$

The equation of a parabola with vertex at (0, 0), focus at (a, 0), and directrix $x = -a$, $a > 0$, is

$$y^2 = 4ax$$

EXAMPLE

Finding the Equation of a Parabola and Graphing It

Find an equation of the parabola with vertex at (0, 0) and focus at (4, 0). Graph the equation.

$$y^2 = 4ax$$

EXAMPLE

Graphing a Parabola Using a Graphing Utility

Graph the parabola $y^2 = 16x$.

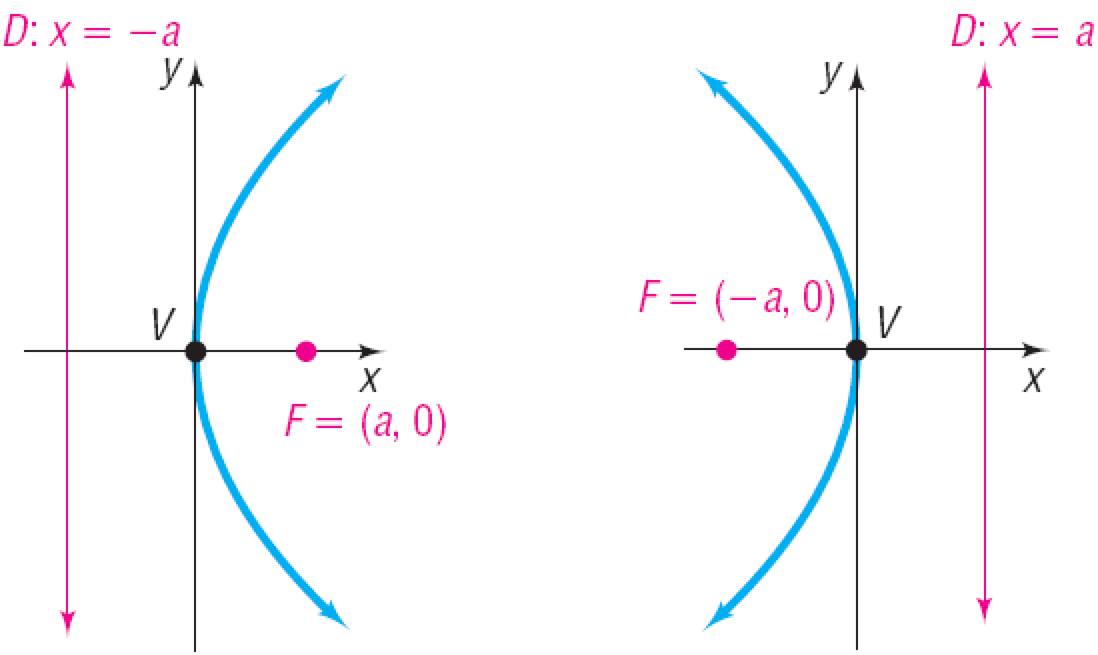
EXAMPLE

Analyzing the Equation of a Parabola

Analyze the equation $y^2 = 10x$.

EQUATIONS OF A PARABOLA VERTEX AT (0, 0); FOCUS ON AN AXIS; $a > 0$

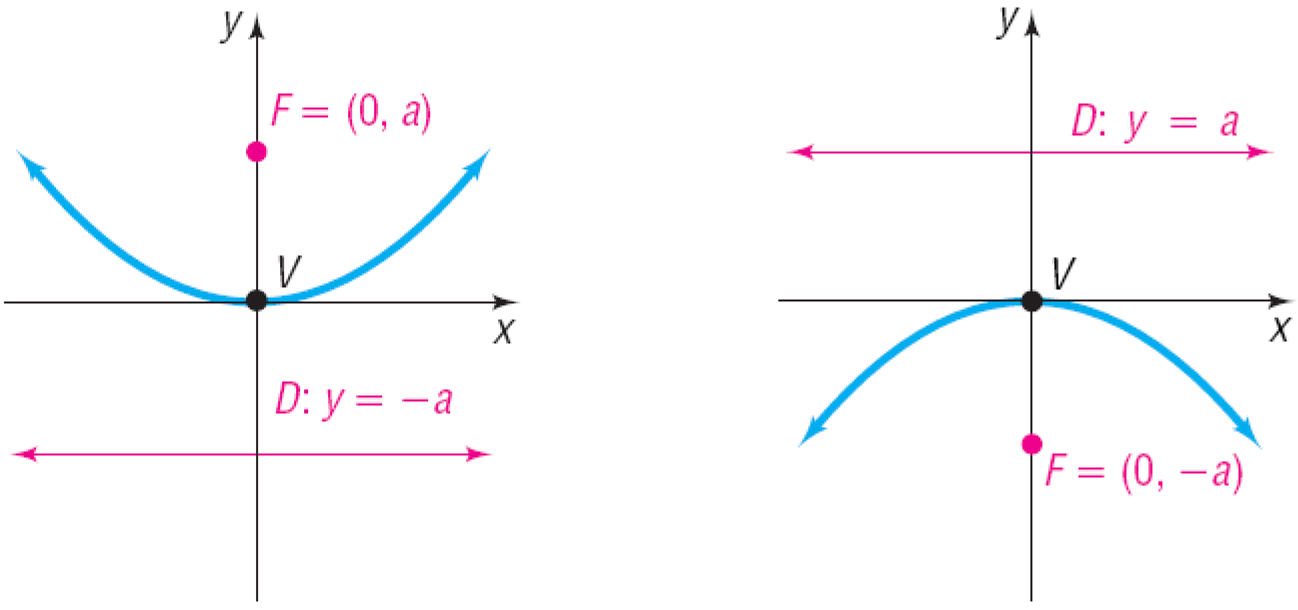
Vertex	Focus	Directrix	Equation	Description
$(0, 0)$	$(a, 0)$	$x = -a$	$y^2 = 4ax$	Parabola, axis of symmetry is the x-axis, opens right
$(0, 0)$	$(-a, 0)$	$x = a$	$y^2 = -4ax$	Parabola, axis of symmetry is the x-axis, opens left



(a) $y^2 = 4ax$ **(b)** $y^2 = -4ax$

EQUATIONS OF A PARABOLA
VERTEX AT (0, 0); FOCUS ON AN AXIS; $a > 0$

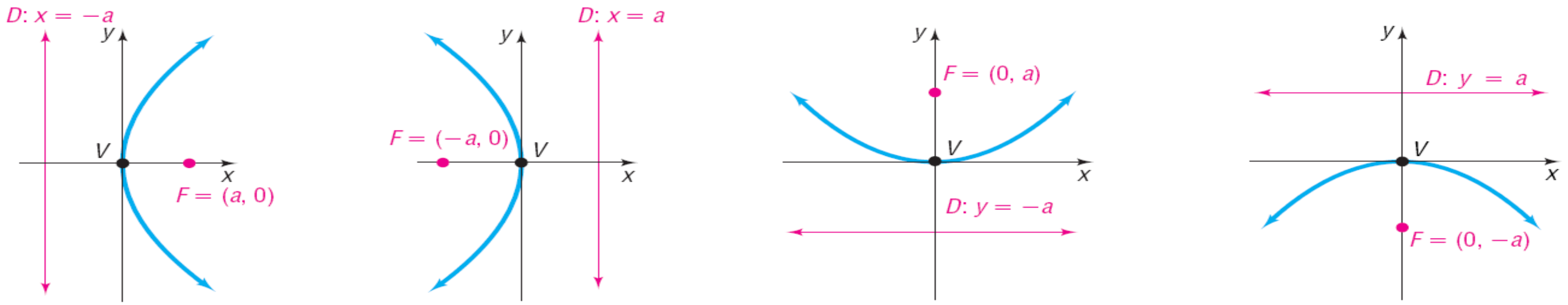
Vertex	Focus	Directrix	Equation	Description
$(0, 0)$	$(0, a)$	$y = -a$	$x^2 = 4ay$	Parabola, axis of symmetry is the y-axis, opens up
$(0, 0)$	$(0, -a)$	$y = a$	$x^2 = -4ay$	Parabola, axis of symmetry is the y-axis, opens down



(c) $x^2 = 4ay$ **(d)** $x^2 = -4ay$

EQUATIONS OF A PARABOLA
VERTEX AT (0, 0); FOCUS ON AN AXIS; $a > 0$

Vertex	Focus	Directrix	Equation	Description
$(0, 0)$	$(a, 0)$	$x = -a$	$y^2 = 4ax$	Parabola, axis of symmetry is the x-axis, opens right
$(0, 0)$	$(-a, 0)$	$x = a$	$y^2 = -4ax$	Parabola, axis of symmetry is the x-axis, opens left
$(0, 0)$	$(0, a)$	$y = -a$	$x^2 = 4ay$	Parabola, axis of symmetry is the y-axis, opens up
$(0, 0)$	$(0, -a)$	$y = a$	$x^2 = -4ay$	Parabola, axis of symmetry is the y-axis, opens down



(a) $y^2 = 4ax$ **(b)** $y^2 = -4ax$ **(c)** $x^2 = 4ay$ **(d)** $x^2 = -4ay$

EXAMPLE

Analyzing the Equation of a Parabola

Analyze the equation $x^2 = -8y$.

EXAMPLE

Finding the Equation of a Parabola

Find the equation of the parabola with focus at (0, – 12) and directrix the line $y = 12$. Graph the equation.

EXAMPLE

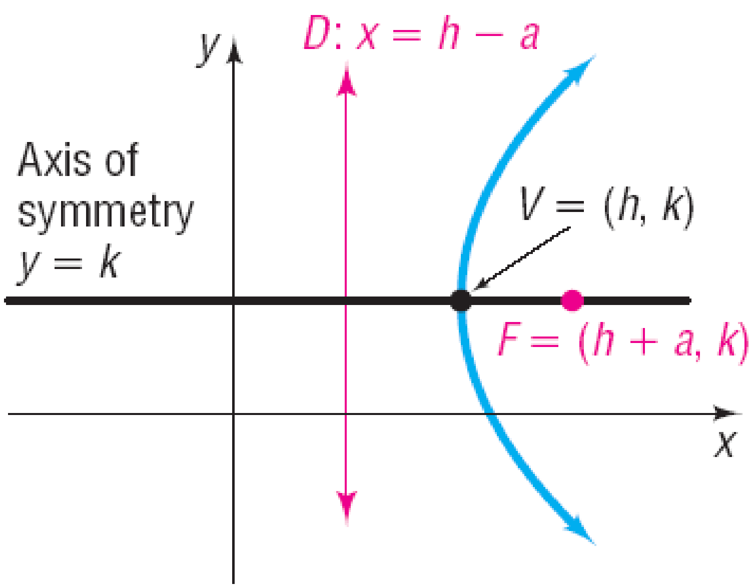
Finding the Equation of a Parabola

Find the equaton of a parabola with vertex at (0, 0) if its axis of symmetry is the y-axis and its graph contains the point $\left(-1,\ -4\right)$. Find its focus and directrix, and graph the equation.

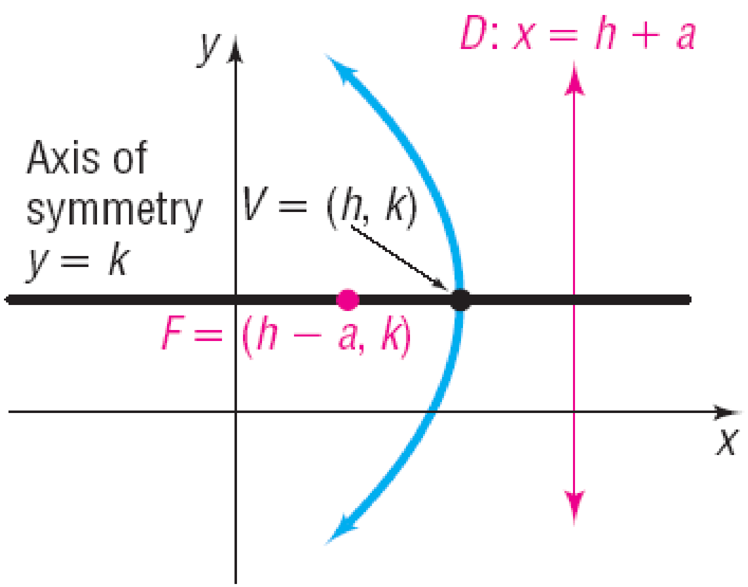
OBJECTIVE 2

2 Analyze Parabolas with Vertex at (h, k)

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h + a, k)$	$x = h - a$	$(y - k)^2 = 4a(x - h)$	Parabola, axis of symmetry parallel to x-axis, opens right
(h, k)	$(h - a, k)$	$x = h + a$	$(y - k)^2 = -4a(x - h)$	Parabola, axis of symmetry parallel to x-axis, opens left

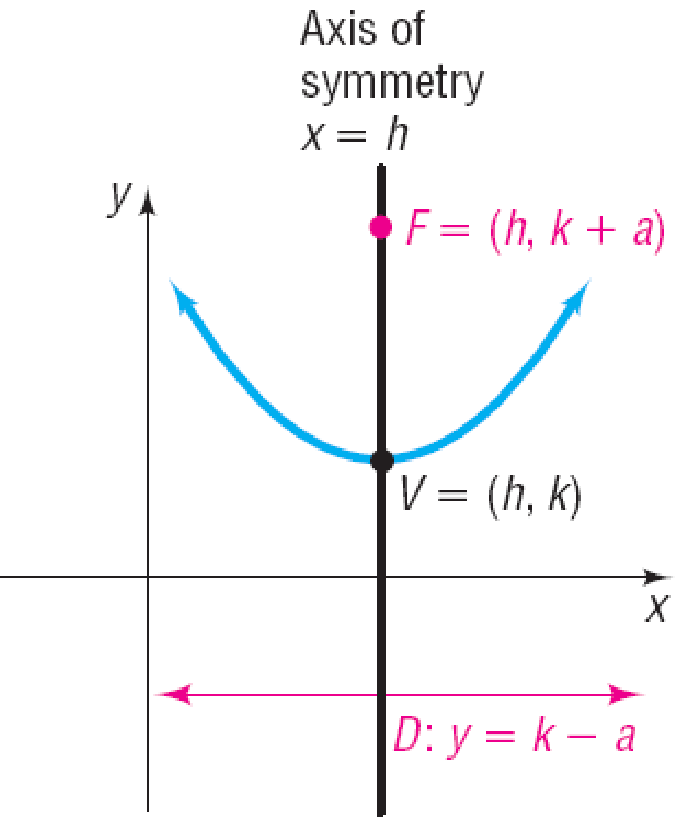


(a) $(y - k)^2 = 4a(x - h)$

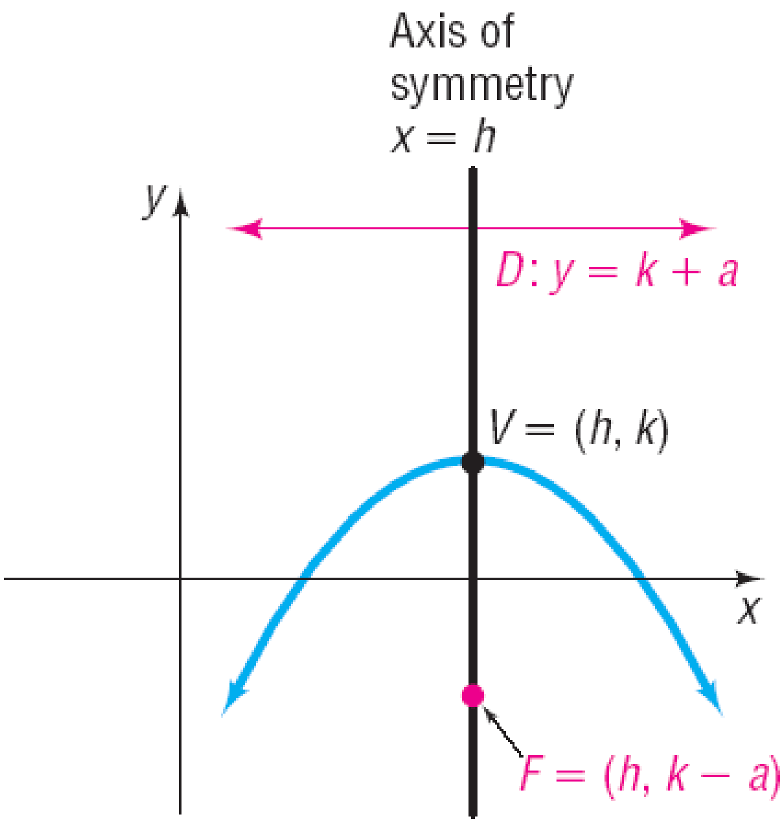


(b) $(y - k)^2 = -4a(x - h)$

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h, k + a)$	$y = k - a$	$(x - h)^2 = 4a(y - k)$	Parabola, axis of symmetry parallel to y-axis, opens up
(h, k)	$(h, k - a)$	$y = k + a$	$(x - h)^2 = -4a(y - k)$	Parabola, axis of symmetry parallel to y-axis, opens down

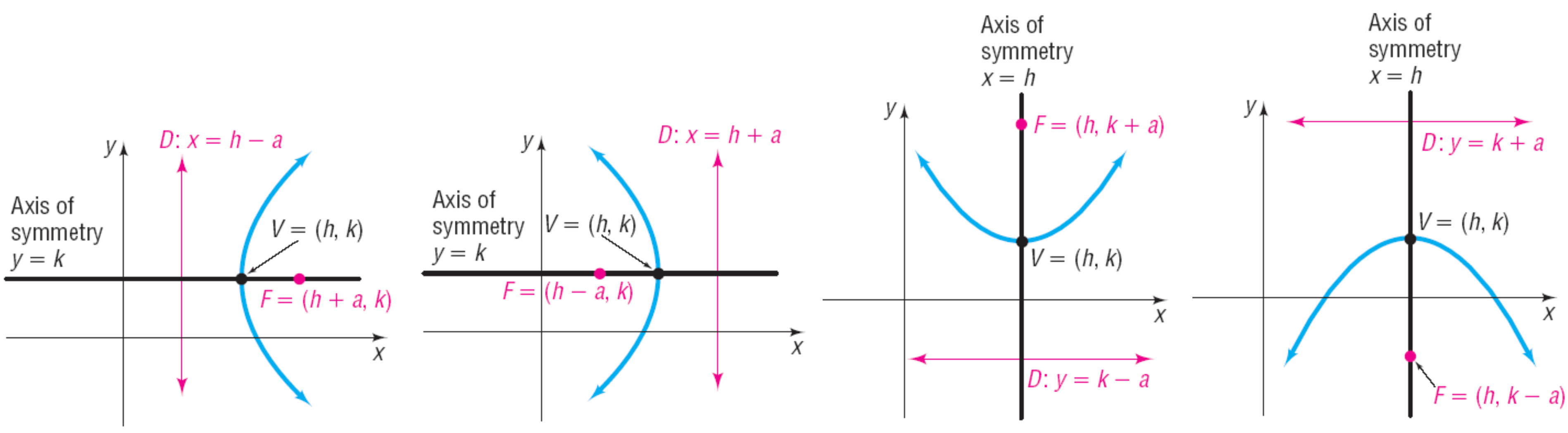


(c) $(x - h)^2 = 4a(y - k)$



(d) $(x - h)^2 = -4a(y - k)$

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h + a, k)$	$x = h - a$	$(y - k)^2 = 4a(x - h)$	Parabola, axis of symmetry parallel to x-axis, opens right
(h, k)	$(h - a, k)$	$x = h + a$	$(y - k)^2 = -4a(x - h)$	Parabola, axis of symmetry parallel to x-axis, opens left
(h, k)	$(h, k + a)$	$y = k - a$	$(x - h)^2 = 4a(y - k)$	Parabola, axis of symmetry parallel to y-axis, opens up
(h, k)	$(h, k - a)$	$y = k + a$	$(x - h)^2 = -4a(y - k)$	Parabola, axis of symmetry parallel to y-axis, opens down



(a) $(y - k)^2 = 4a(x - h)$ **(b)** $(y - k)^2 = -4a(x - h)$ **(c)** $(x - h)^2 = 4a(y - k)$ **(d)** $(x - h)^2 = -4a(y - k)$

EXAMPLE

Finding the Equation of a Parabola, Vertex Not at the Origin

Find an equation of the parabola with vertex at (5, – 1) and focus at (2, – 1). Graph the equation.

EXAMPLE

Using a Graphing Utility to Graph a Parabola, Vertex Not at Origin

Using a graphing utility, graph the equation $(y+2)^2 = 12(x-4)$

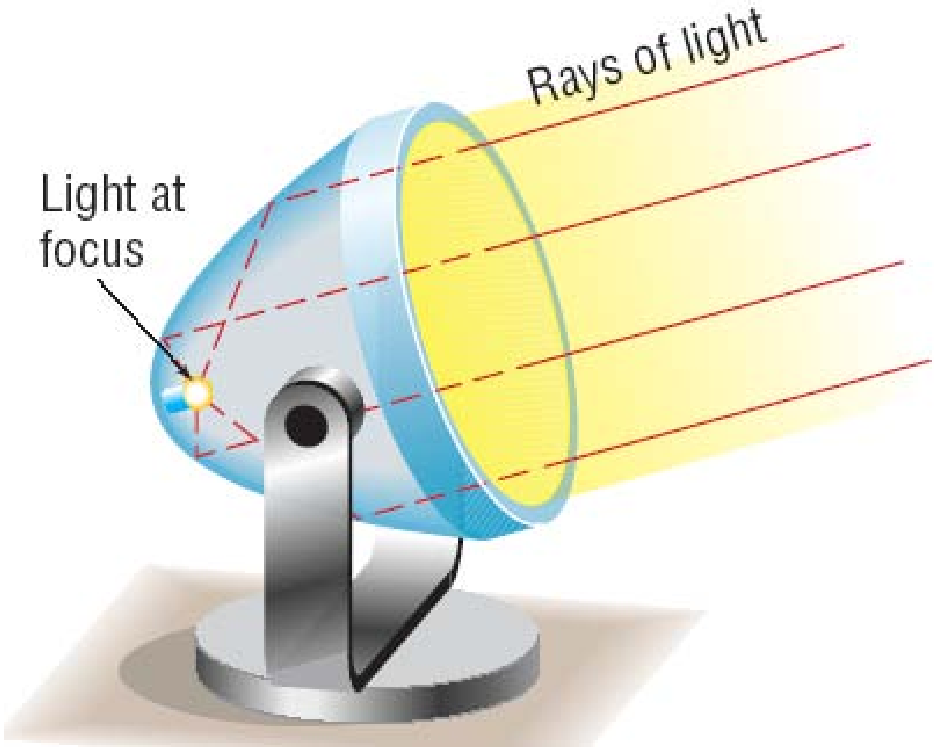
EXAMPLE

Analyzing the Equation of a Parabola

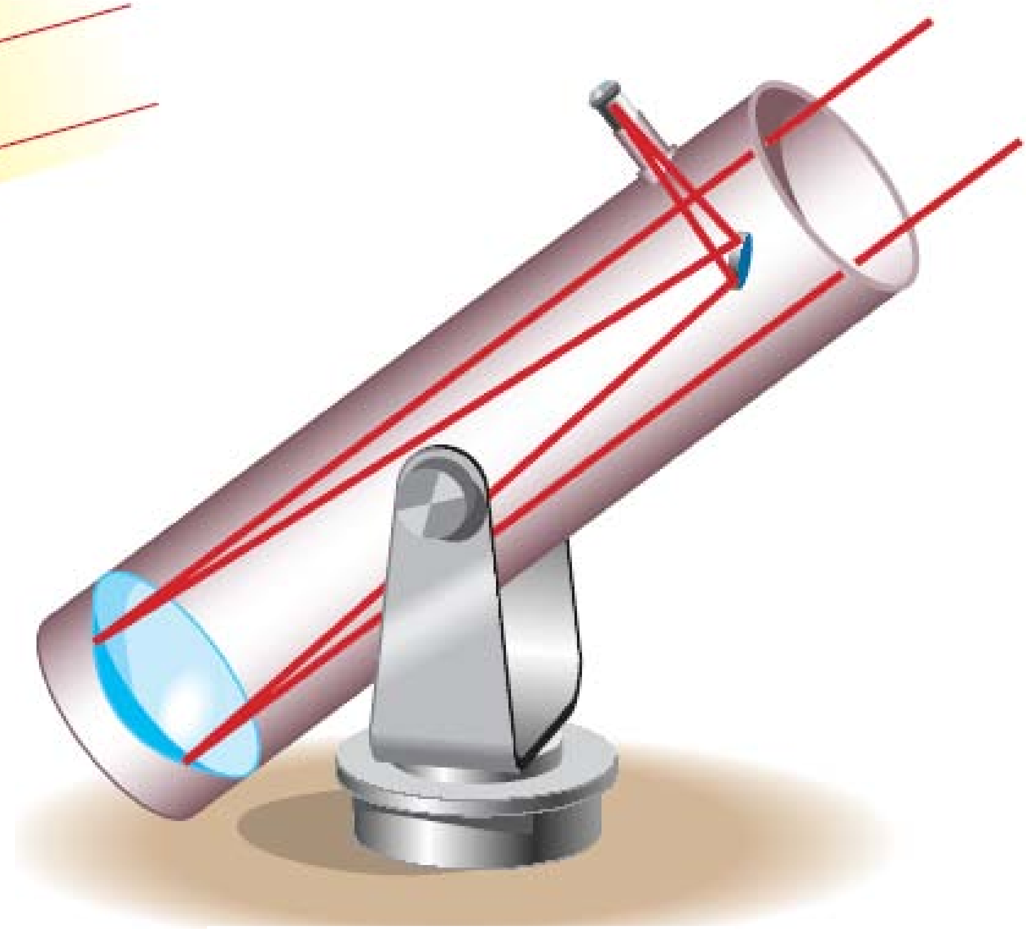
Analyze the equation: $y^2 - 4x + 4y = 0$

OBJECTIVE 3

3 Solve Applied Problems Involving Parabolas



Searchlight



Telescope

EXAMPLE Satellite Dish

A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 10 ft across at its opening and 3 feet deep at its center, at what position should the receiver be placed?