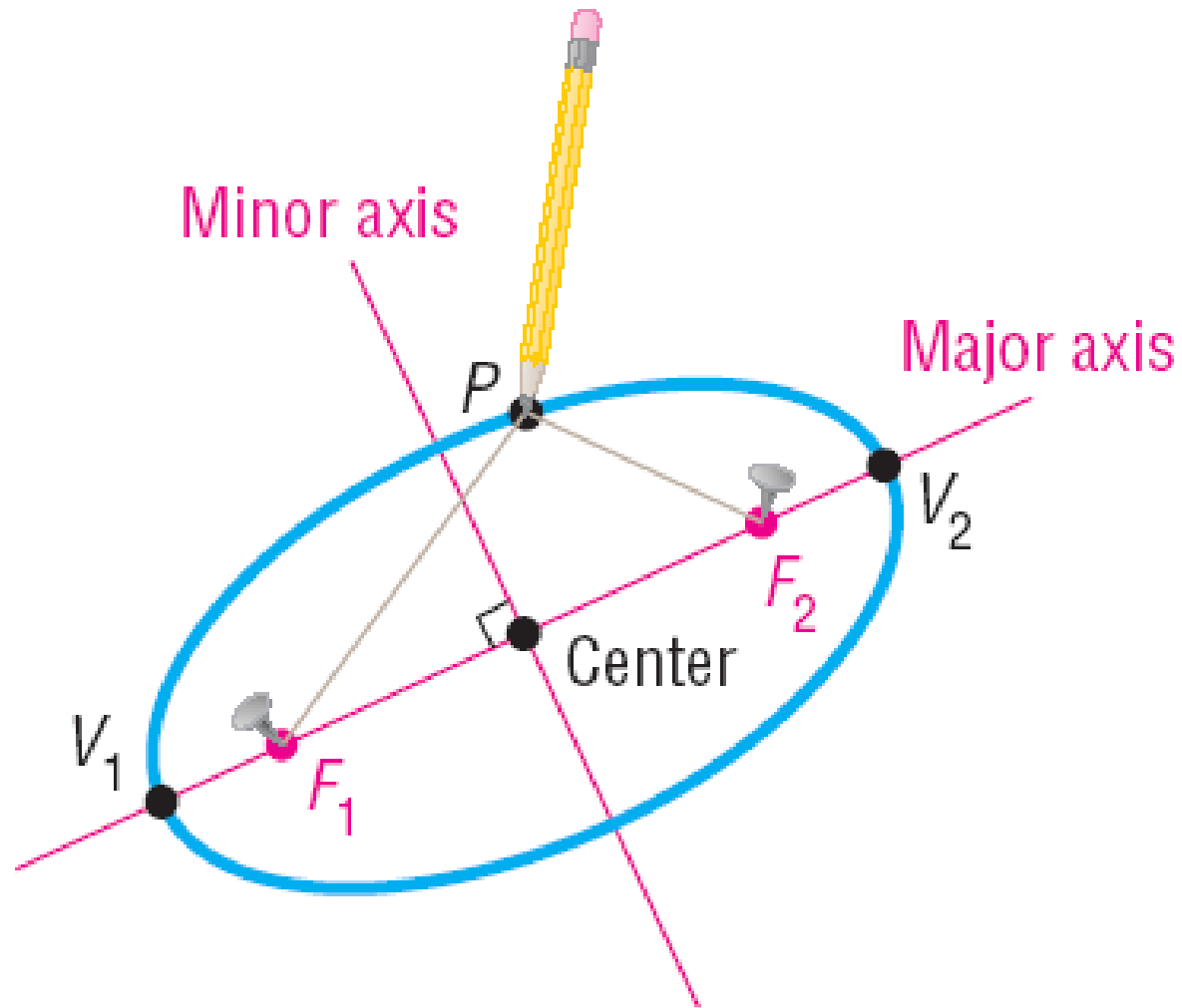


Section 11.3

The Ellipse

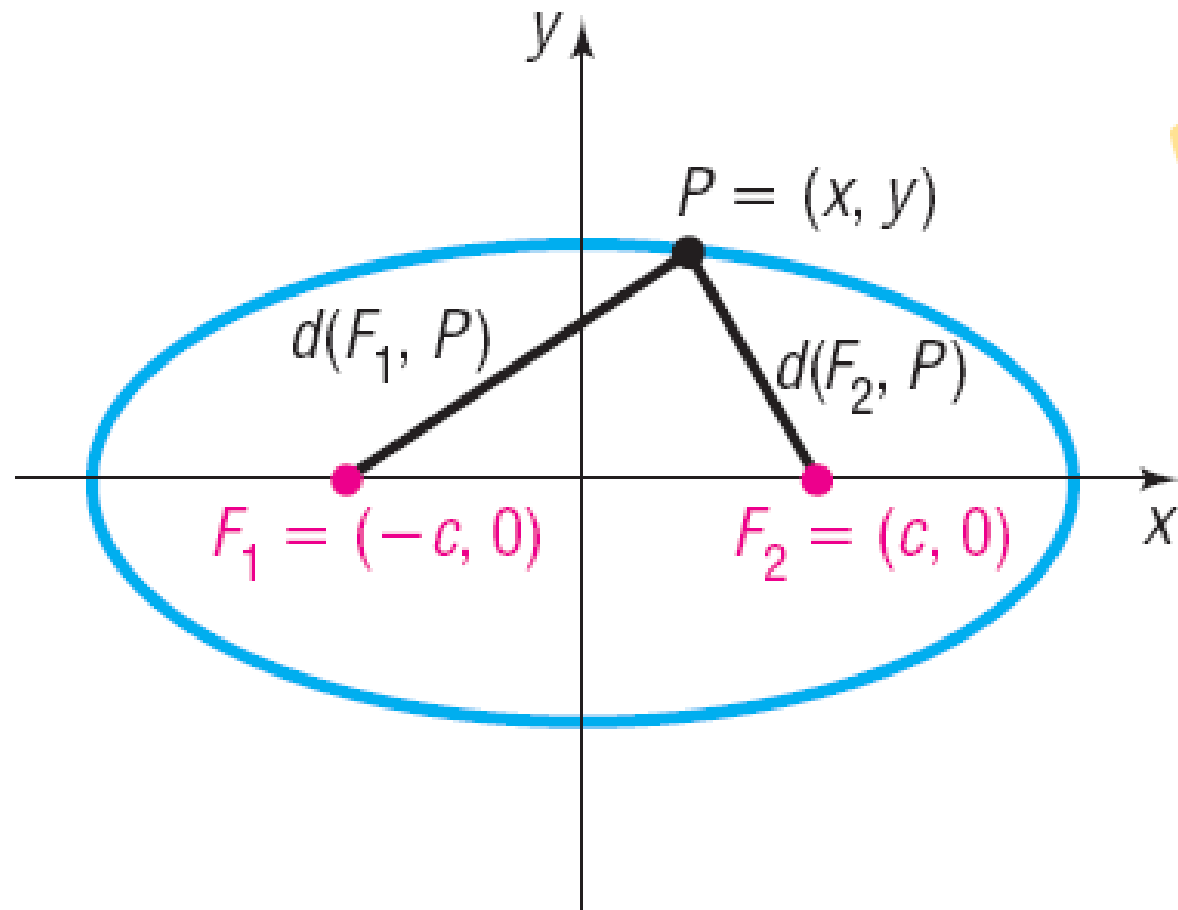
An **ellipse** is the collection of all points in the plane the sum of whose distances from two fixed points, called the **foci**, is a constant.



OBJECTIVE 1

- 1 ✓ Analyze Ellipses with Center at the Origin

$$d(F_1, P) + d(F_2, P) = 2a$$



Equation of an Ellipse

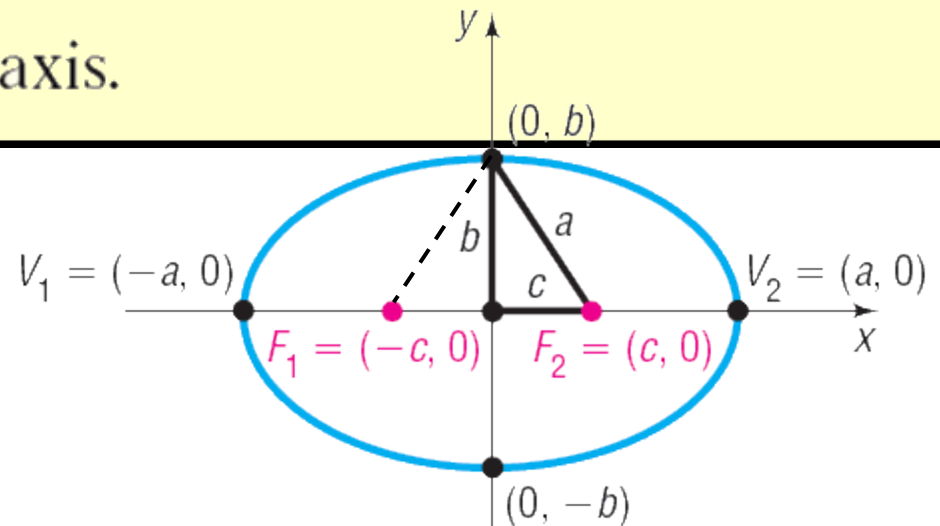
Center at $(0, 0)$; Major Axis along the x -Axis

An equation of the ellipse with center at $(0, 0)$,

foci at $(-c, 0)$ and $(c, 0)$, and
vertices at $(-a, 0)$ and $(a, 0)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

The major axis is the x -axis.



EXAMPLE**Finding an Equation of an Ellipse**

Find an equation of the ellipse with center at the origin, one focus at $(0, -3)$ and a vertex at $(5, 0)$. Graph the equation.

EXAMPLE**Graphing an Ellipse Using a Graphing Utility**

Use a graphing utility to graph the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$

EXAMPLE

Analyzing the Equation of an Ellipse

Analyze the equation: $\frac{x^2}{16} + \frac{y^2}{8} = 1$

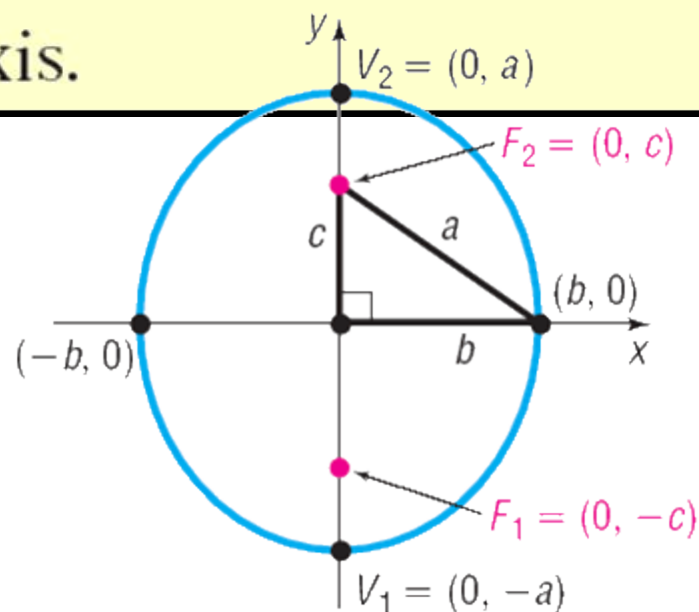
Equation of an Ellipse

Center at (0, 0); Major Axis along the y-Axis

An equation of the ellipse with center at (0, 0),
foci at (0, -c) and (0, c), and
vertices at (0, -a) and (0, a) is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

The major axis is the y-axis.



EXAMPLE

Analyzing the Equation of an Ellipse

Analyze the equation $\frac{x^2}{9} + \frac{y^2}{4} = 36$

EXAMPLE

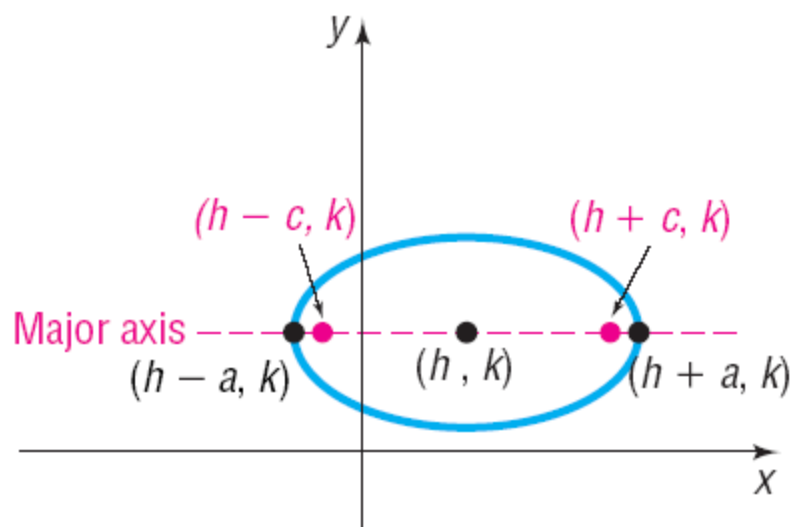
Finding an Equation of an Ellipse

Find an equation of the ellipse having one focus at $(0, 2)$ and vertices at $(0, -5)$ and $(0, 5)$. Graph the equation by hand.

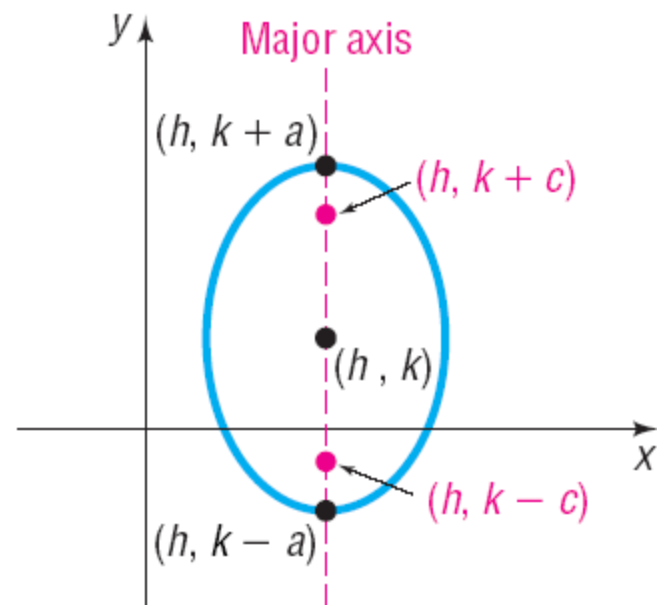
OBJECTIVE 2

- 2 Analyze Ellipses with Center at (h, k)

Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to x -axis	$(h + c, k)$ $(h - c, k)$	$(h + a, k)$ $(h - a, k)$	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1,$ $a > b$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to y -axis	$(h, k + c)$ $(h, k - c)$	$(h, k + a)$ $(h, k - a)$	$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1,$ $a > b$ and $b^2 = a^2 - c^2$



(a) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$



(b) $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$

EXAMPLE

Finding an Equation of an Ellipse, Center Not at the Origin

Find an equation for the ellipse with center at $(-2, 4)$, one focus at $(3, 4)$, and one vertex at $(-7, 4)$. Graph the equation by hand.

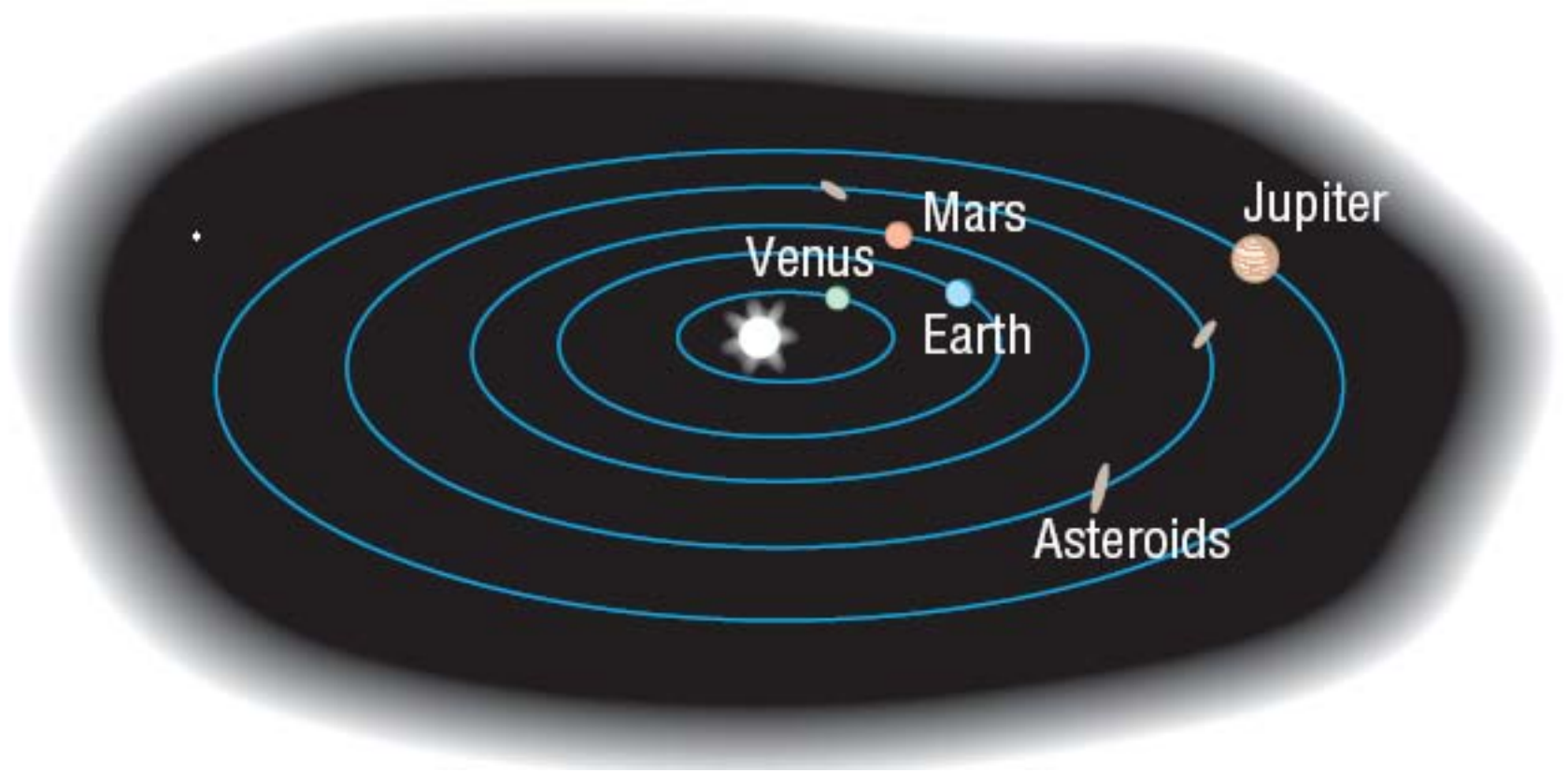
EXAMPLE

Analyzing the Equation of an Ellipse

Analyze the equation: $25x^2 + 4y^2 + 150x + 16y + 91 = 0$

OBJECTIVE 3

- 3 Solve Applied Problems Involving Ellipses



EXAMPLE

A Whispering Gallery



The whispering gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center?