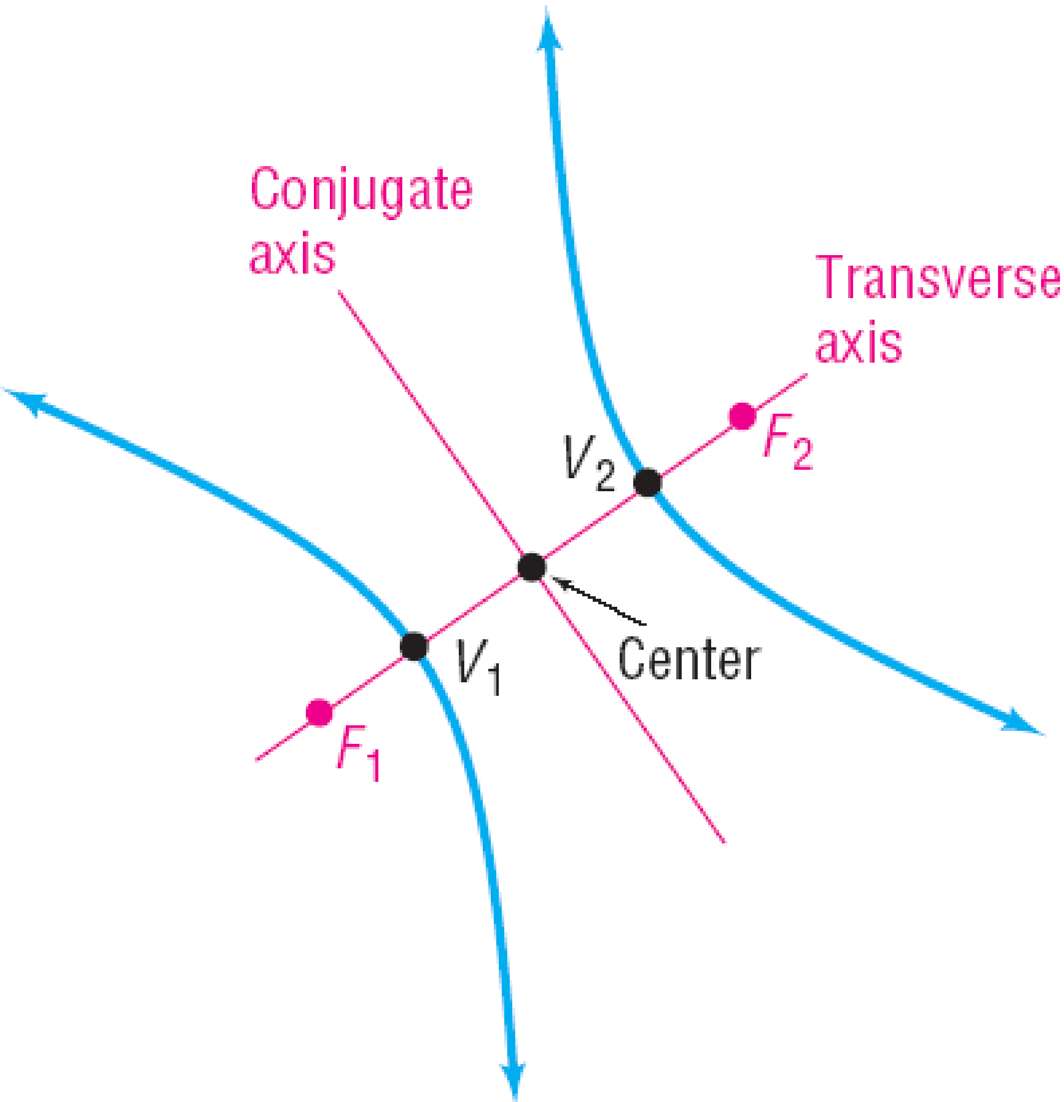
Section 11.4

The Hyperbola

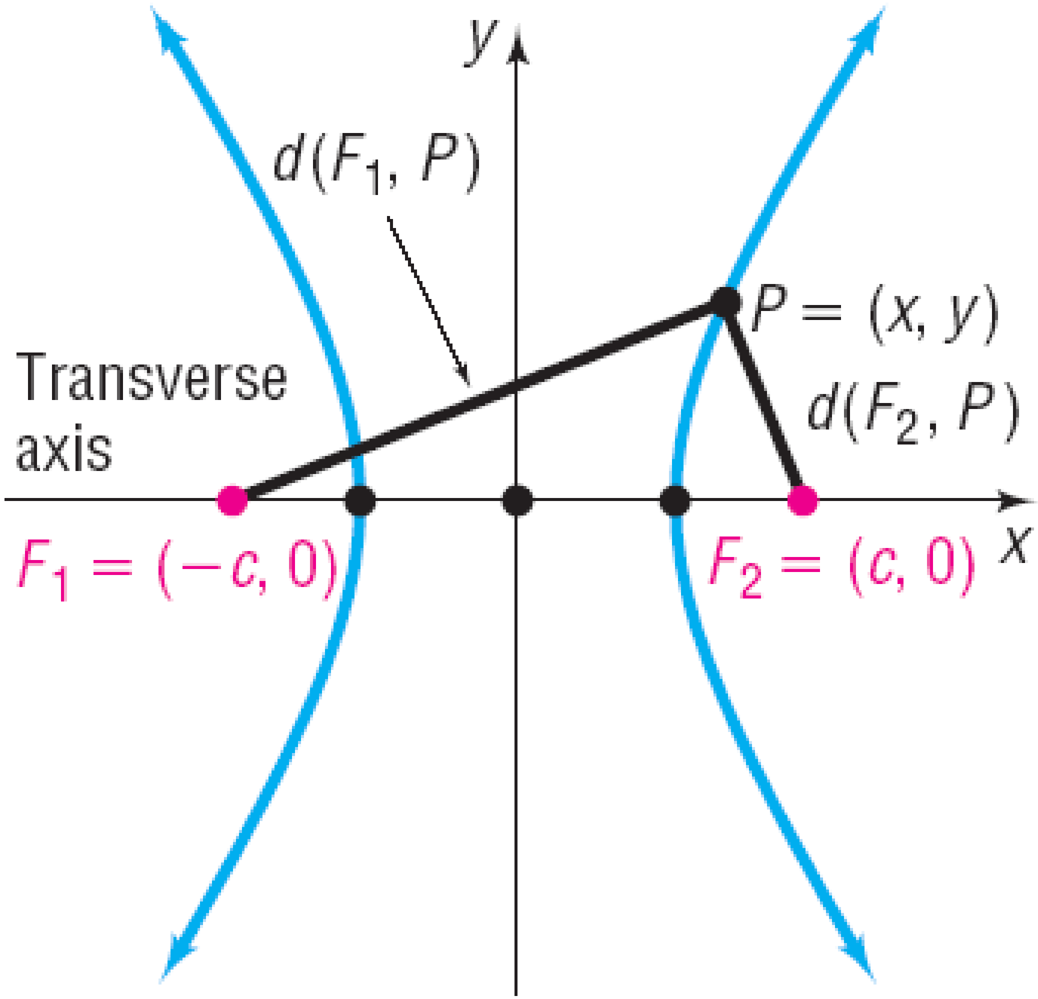
A **hyperbola** is the collection of all points in the plane the difference of whose distances from two fixed points, called the **foci**, is a constant.



OBJECTIVE 1

1 Analyze Hyperbolas with Center at the Origin

$$d(F_1, P) - d(F_2, P) = \pm 2a$$

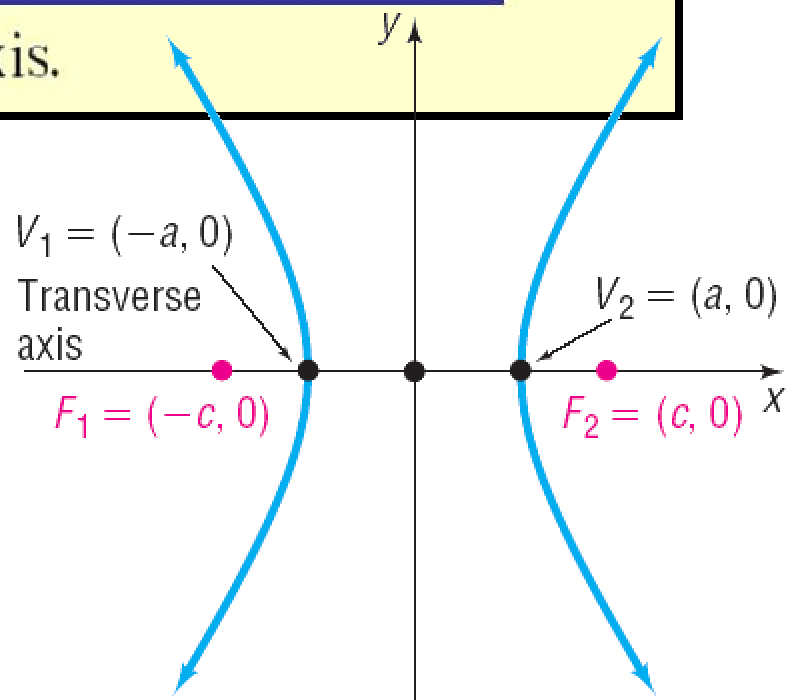


Equation of a Hyperbola Center at (0, 0) Transverse Axis along the x-Axis

An equation of the hyperbola with center at $(0, 0)$, foci at $(-c, 0)$ and $(c, 0)$, and vertices at $(-a, 0)$ and $(a, 0)$ is

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad \text{where } b^2 = c^2 - a^2}$$

The transverse axis is the x-axis.



EXAMPLE

Finding and Graphing an Equation of a Hyperbola

Find an equation of the hyperbola with center at the origin, one focus at (– 5, 0), and one vertex at (2, 0). Graph the equation.

EXAMPLE

Using a Graphing Utility to Graph a Hyperbola

Use a graphing utility to graph the ellipse $\dfrac{x^2}{36} - \dfrac{y^2}{25} = 1$

EXAMPLE

Analyzing the Equation of a Hyperbola

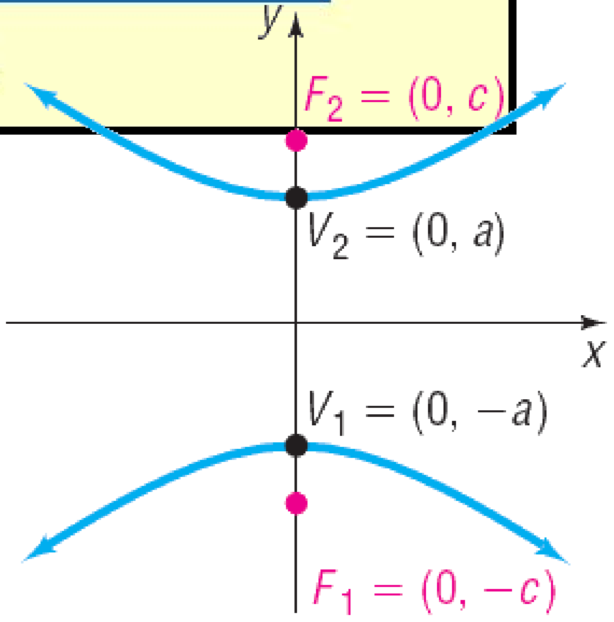
Analyze the equation $\frac{x^2}{25}-\frac{y^2}{16}=1$

Equation of a Hyperbola; Center at (0, 0); Transverse Axis along the y-Axis

An equation of the hyperbola with center at (0, 0), foci at $(0, -c)$ and $(0, c)$, and vertices at $(0, -a)$ and $(0, a)$ is

$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1, \qquad \text{where } b^2 = c^2 - a^2}$$

The transverse axis is the y-axis.



EXAMPLE

Analyzing the Equation of a Hyperbola

Analyze the equation $2y^2 - 8x^2 = 32$

EXAMPLE

Finding an Equation of a Hyperbola

Find an equation of the hyperbola having one vertex at (0, 4) and foci at (0, – 7) and (0, 7). Graph the equation.

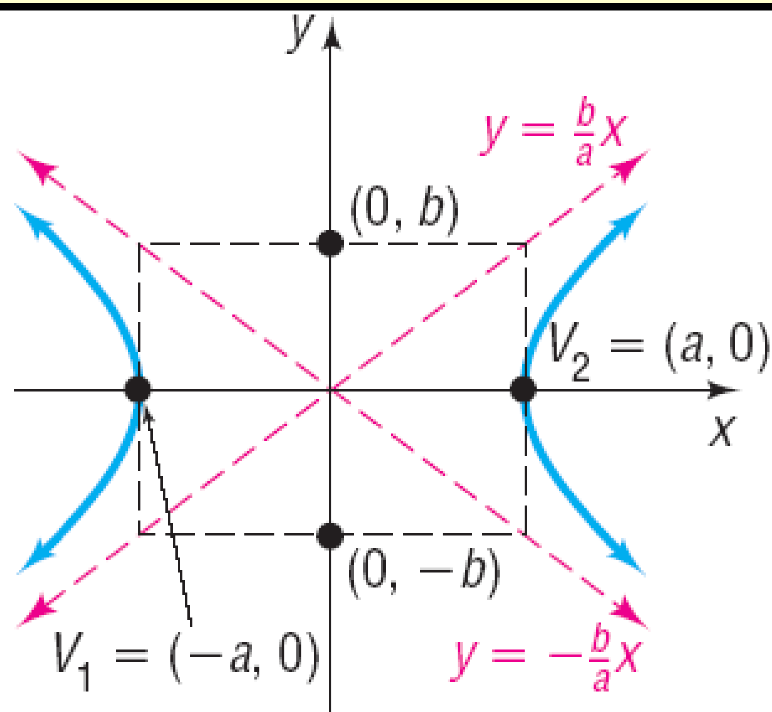
OBJECTIVE 2

2 Find the Asymptotes of a Hyperbola

Asymptotes of a Hyperbola

The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has the two oblique asymptotes

$$\boxed{y = \frac{b}{a}x \quad \text{and} \quad y = -\frac{b}{a}x}$$



Asymptotes of a Hyperbola

The hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ has the two oblique asymptotes

$$\boxed{y = \frac{a}{b}x \quad \text{and} \quad y = -\frac{a}{b}x}$$

EXAMPLE

Analyzing the Equation of a Hyperbola

Analyze the equation $4x^2 - y^2 = 16$

Seeing the Concept

Refer to Figure 44(b). Create a TABLE using Y_1 and Y_4 with $x = 10, 100, 1000,$ and $10{,}000$. Compare the values of Y_1 and Y_4. Repeat for Y_1 and Y_3, Y_2 and Y_3, and Y_2 and Y_4.

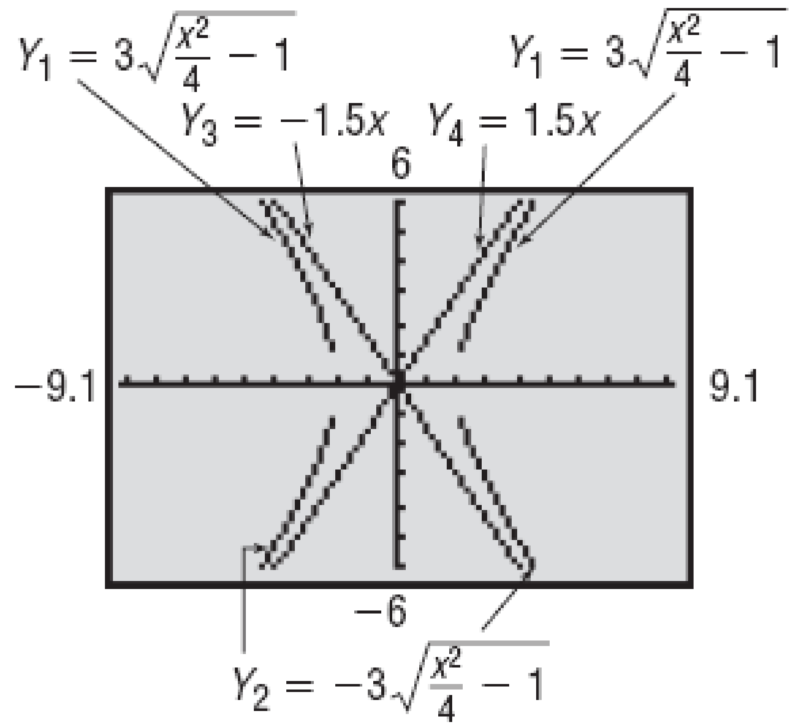


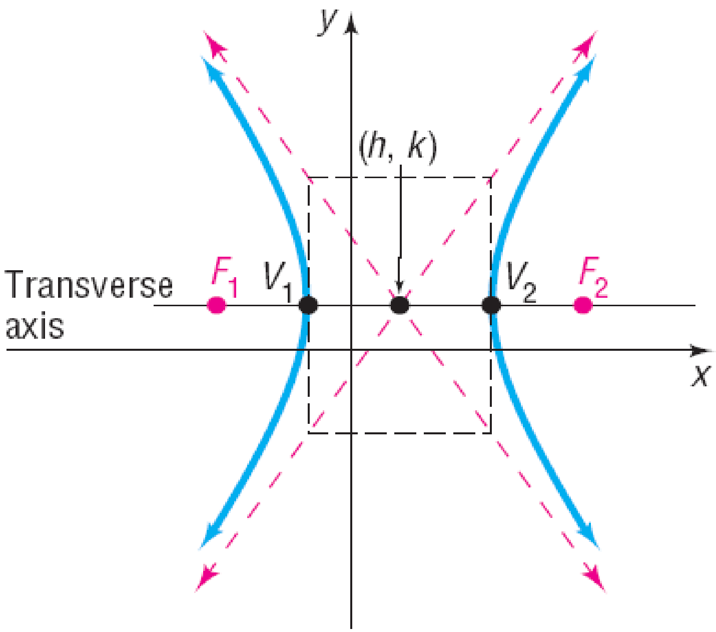
Figure 44 **(b)**

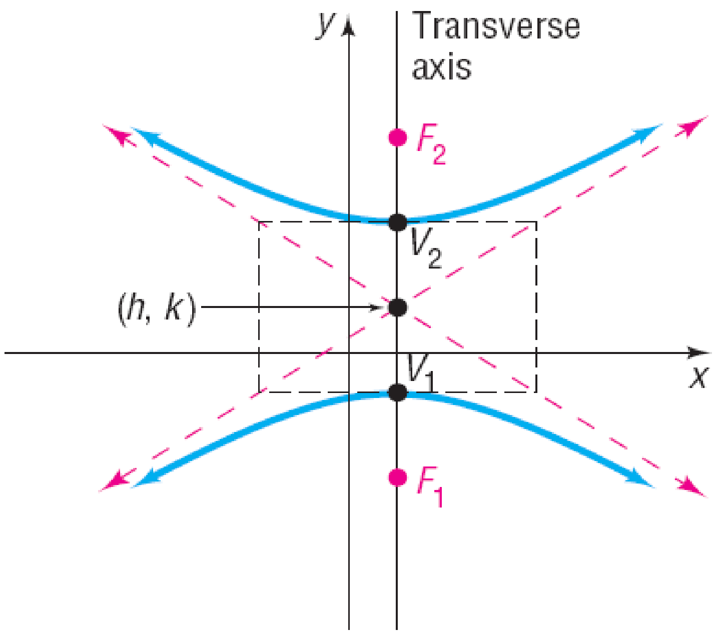
OBJECTIVE 3

3 Analyze Hyperbolas with Center at (h, k)

HYPERBOLAS WITH CENTER AT (h, k) AND TRANSVERSE AXIS PARALLEL TO A COORDINATE AXIS

Center	Transverse Axis	Foci	Vertices	Equation	Asymptotes
(h, k)	Parallel to the x-axis	$(h \pm c, k)$	$(h \pm a, k)$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1,\quad b^2 = c^2 - a^2$	$y - k = \pm\frac{b}{a}(x - h)$
(h, k)	Parallel to the y-axis	$(h, k \pm c)$	$(h, k \pm a)$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1,\quad b^2 = c^2 - a^2$	$y - k = \pm\frac{a}{b}(x - h)$





EXAMPLE

Finding an Equation of a Hyperbola, Center Not at the Origin

Find an equation for the hyperbola with center at (–1, 3), one focus at (– 1, – 1), and one vertex at (– 1, 4). Graph the equation by hand.

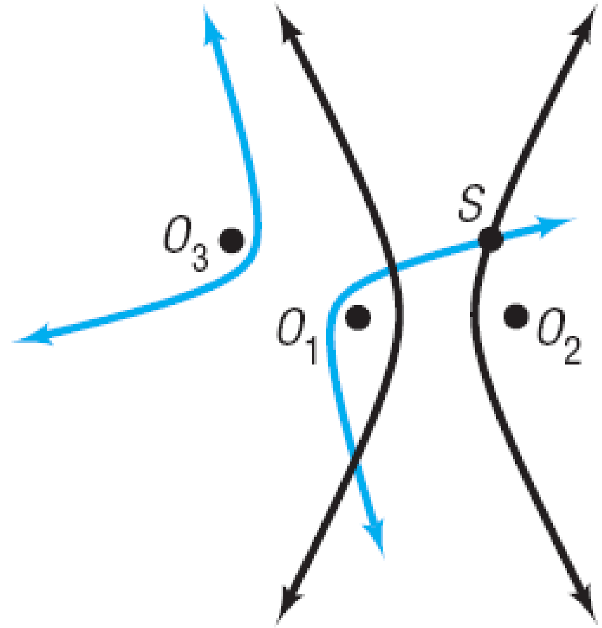
EXAMPLE

Analyzing the Equation of a Hyperbola

Analyze the equation $-16x^2 + y^2 - 64x - 6y - 71 = 0$

OBJECTIVE 4

4 Solve Applied Problems Involving Hyperbolas



Look at Figure 48. Suppose that three microphones are located at points O_1, O_2, and O_3 (the foci of the two hyperbolas). In addition, suppose that a gun is fired at S and the microphone at O_1 records the gun shot 1 second after the microphone at O_2. Because sound travels at about 1100 feet per second, we conclude that the microphone at O_1 is 1100 feet farther from the gunshot than O_2. We can model this situation by saying that S lies on the same branch of a hyperbola with foci at O_1 and O_2. (Do you see why? The difference of the distances from S to O_1 and from S to O_2 is the constant 1100.) If the third microphone at O_3 records the gunshot 2 seconds after O_1, then S will lie on a branch of a second hyperbola with foci at O_1 and O_3. In this case, the constant difference will be 2200. The intersection of the two hyperbolas will identify the location of S.

EXAMPLE Lightning Strikes

Suppose that two people standing 1 mile apart both see a flash of lightning. After a period of time, the person standing at point A hears the thunder. One second later, the person standing at point B hears the thunder. If the person at B is due west of the person at A and the lightning strike is known to occur due north of the person standing at point A, where did the lightning strike?