Section 12.1

Systems of Linear Equations;

Substitution and Elimination

EXAMPLE Movie Theater Ticket Sales

A movie theater sells tickets for $9.00 each, with seniors receiving a discount of $2.00. One evening the theater took in $4760 in revenue. If x represents the number of tickets sold at $9.00 and y the number of tickets sold at the discounted price of $7.00, write an equation that relates these variables.

Suppose that we also know that 600 tickets were sold that evening. Can you write another equation relating the variable x and y?

EXAMPLE Examples of Systems of Equations

(a) $$\begin{cases} 2x + y = 5 & (1) \\ -4x + 6y = -2 & (2) \end{cases}$$ Two equations containing two variables, x and y

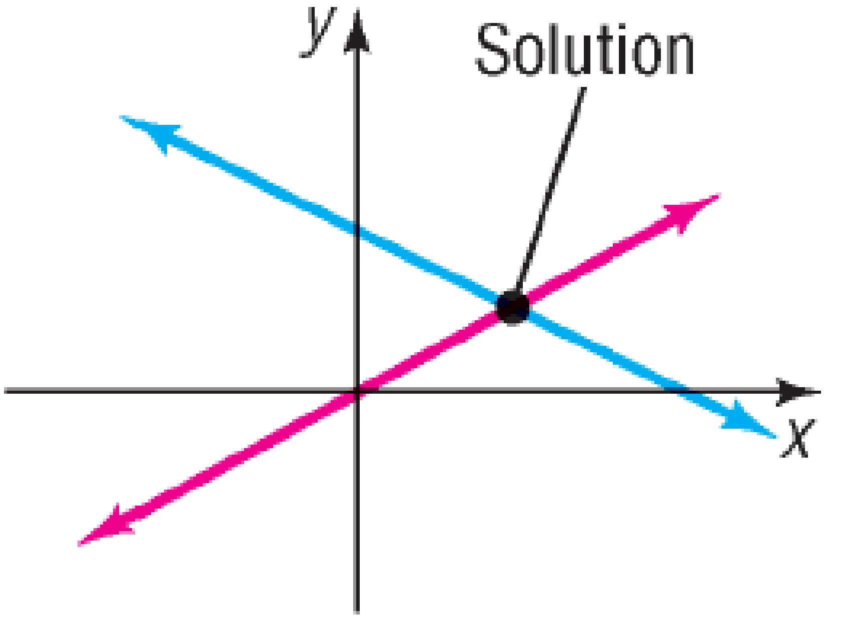
(b) $$\begin{cases} x + y^2 = 5 & (1) \\ 2x + y = 4 & (2) \end{cases}$$ Two equations containing two variables, x and y

(c) $$\begin{cases} x + y + z = 6 & (1) \\ 3x - 2y + 4z = 9 & (2) \\ x - y - z = 0 & (3) \end{cases}$$ Three equations containing three variables, x, y, and z

(d) $$\begin{cases} x + y + z = 5 & (1) \\ x - y = 2 & (2) \end{cases}$$ Two equations containing three variables, x, y, and z

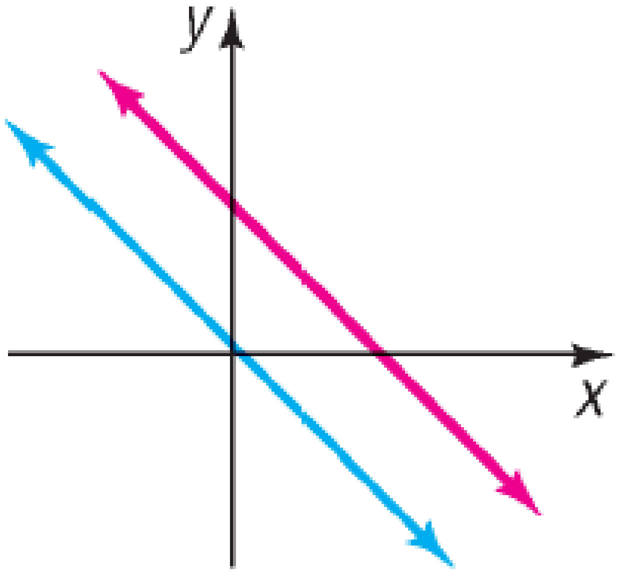
(e) $$\begin{cases} x + y + z = 6 & (1) \\ 2x + 2z = 4 & (2) \\ y + z = 2 & (3) \\ x = 4 & (4) \end{cases}$$ Four equations containing three variables, x, y, and z

1. If the lines intersect, then the system of equations has one solution, given by the point of intersection. The system is **consistent** and the equations are **independent**.



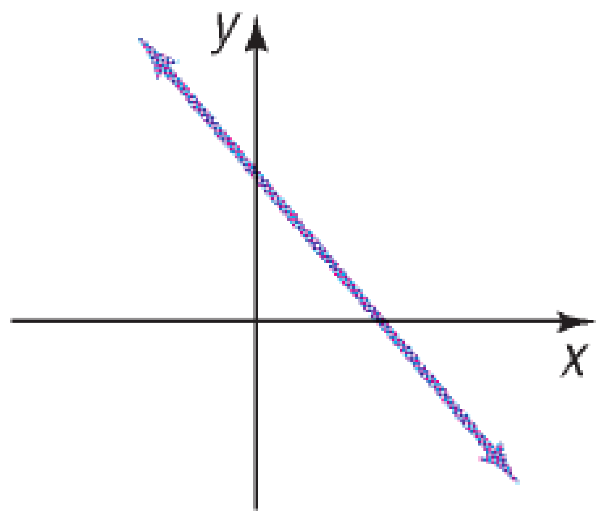
(a) Intersecting lines; system has one solution

2. If the lines are parallel, then the system of equations has no solution, because the lines never intersect. The system is **inconsistent**.



(b) Parallel lines; system has no solution

3. If the lines are coincident, then the system of equations has infinitely many solutions, represented by the totality of points on the line. The system is **consistent** and the equations are **dependent**.



(c) Coincident lines; system has infinitely many solutions

EXAMPLE

Solving a System of Linear Equations Using a Graphing Utility

Solve: $\begin{cases} 2x - y = 13 \\ -4x - 9y = 7 \end{cases}$

OBJECTIVE 1

✓ 1 Solve Systems of Equations by Substitution

EXAMPLE

Solving a System of Linear Equations by Substitution

Solve: $$\begin{cases} 2x - y = 13 \\ -4x - 9y = 7 \end{cases}$$

OBJECTIVE 2

2 Solve Systems of Equations by Elimination

Rules for Obtaining an Equivalent System of Equations

1. Interchange any two equations of the system.
2. Multiply (or divide) each side of an equation by the same nonzero constant.
3. Replace any equation in the system by the sum (or difference) of that equation and a nonzero multiple of any other equation in the system.

EXAMPLE

Solving a System of Linear Equations by Elimination

Solve: $\begin{cases} \dfrac{3}{2}x + \dfrac{y}{8} = -1 \\ 16x + 3y = -28 \end{cases}$

Let's return to the movie theater example

EXAMPLE **Movie Theater Ticket Sales**

A movie theater sells tickets for $9.00 each, with seniors receiving a discount of $2.00. One evening the theater sold 600 tickets and took in $4760 in revenue. How many of each type of ticket were sold?

OBJECTIVE 3

3 Identify Inconsistent Systems of Equations Containing Two Variables

EXAMPLE

An Inconsistent System of Linear Equations

Solve: $\begin{cases} 3x + 6y = 12 \\ x + 2y = 7 \end{cases}$

OBJECTIVE 4

4 Express the Solution of a System of Dependent Equations Containing Two Variables

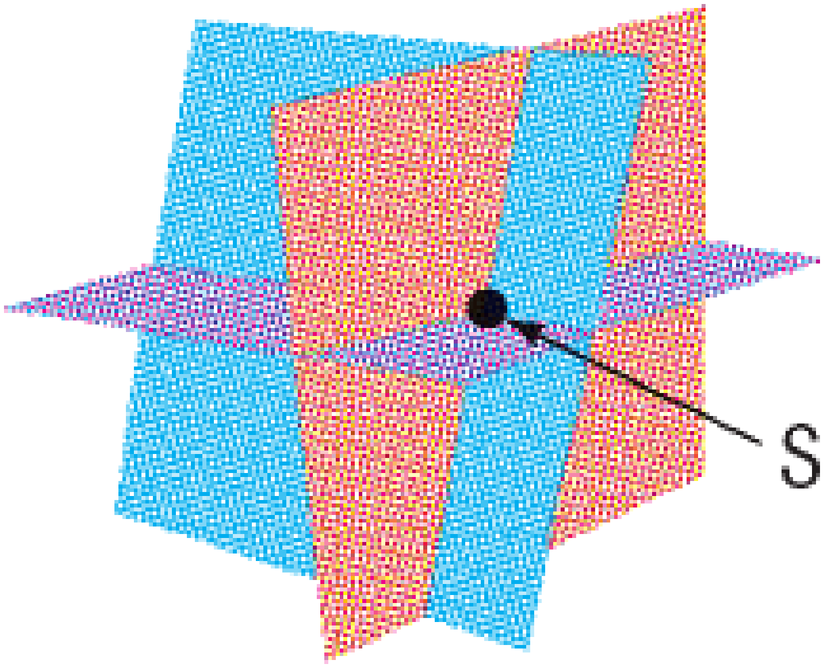
EXAMPLE

Solving a System of Dependent Equations

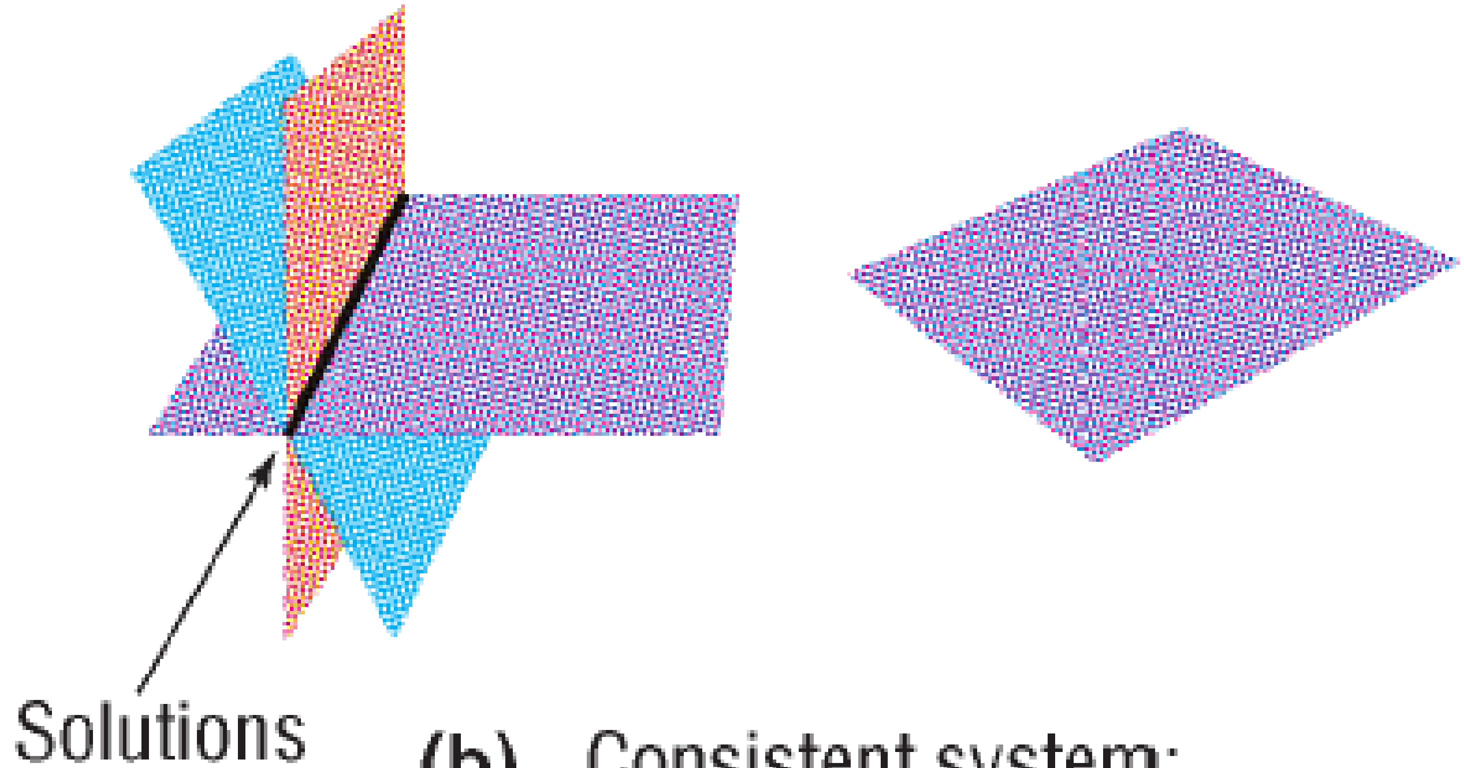
Solve: $\begin{cases} 6x - 4y = 8 \\ -9x + 6y = -12 \end{cases}$

OBJECTIVE 5

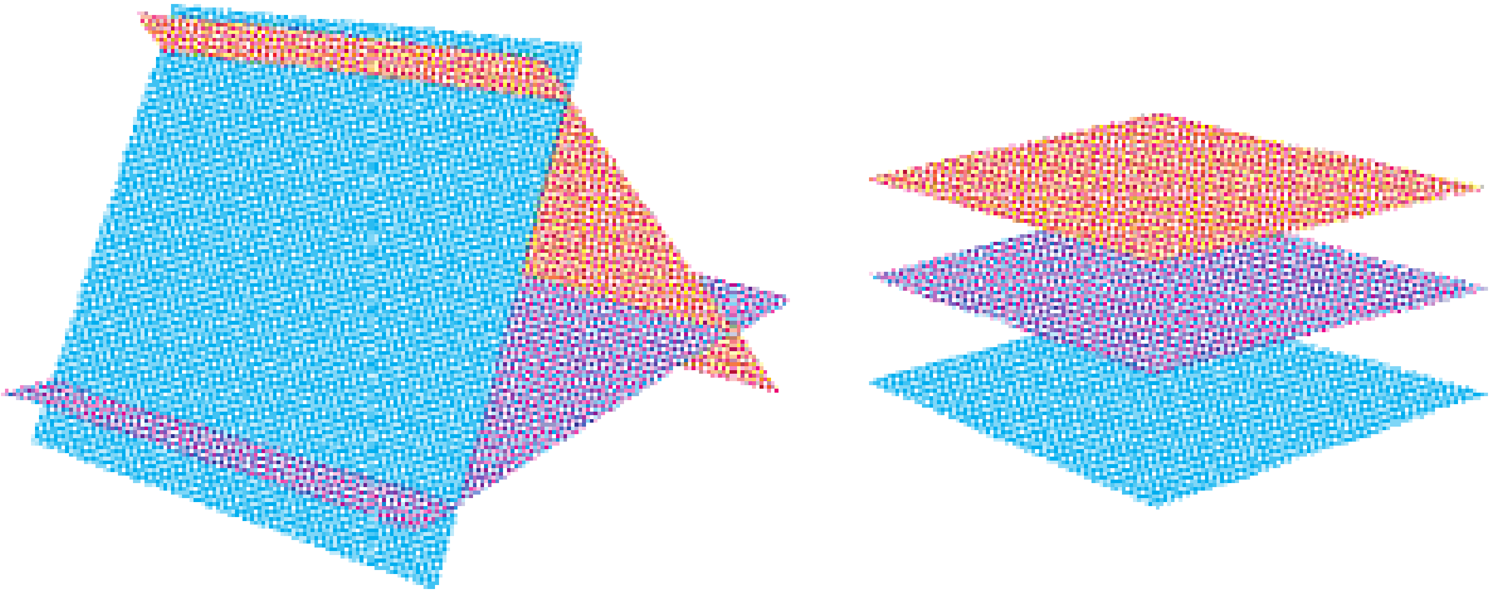
5 Solve Systems of Three Equations Containing Three Variables



(a) Consistent system; one solution



(b) Consistent system; infinite number of solutions



(c) Inconsistent system;
no solution

EXAMPLE

Solving a System of Three Linear Equations with Three Variables

Use the method of elimination to solve the system of equations.

$$\begin{cases} 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \\ x - 2y + 3z = 7 \end{cases}$$

OBJECTIVE 6

6 Identify Inconsistent Systems of Equations Containing Three Variables

EXAMPLE

An Inconsistent System of Linear Equations

Solve: $$\begin{cases} 2x-3y-z=0 \\ -x+2y+z=5 \\ 3x-4y-z=1 \end{cases}$$

OBJECTIVE 7

7 Express the Solution of a System of Dependent Equations Containing Three Variables

EXAMPLE

Solving a System of Dependent Equations

Solve: $$\begin{cases} x+y+2z=1 \\ 2x-y+z=2 \\ 4x+y+5z=4 \end{cases}$$

EXAMPLE **Curve Fitting**

Find real numbers a, b, and c so that the graph of the quadratic function $y = ax^2 + bx + c$ contains the points (– 2, 13), (0, 1) and (– 2, 3).