Section 12.2

Systems of Linear Equations: Matrices

$$\begin{cases} x + 4y = 14 \\ 3x - 2y = 0 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 4 & 14 \\ 3 & -2 & 0 \end{array}\right]$$

A **matrix** is defined as a rectangular array of numbers,

$$
\begin{array}{c|cccccc|}
 & \text{Column } 1 & \text{Column } 2 & & \text{Column } j & & \text{Column } n \\
\text{Row } 1 & a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\
\text{Row } 2 & a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
\text{Row } i & a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
\text{Row } m & a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn}
\end{array}
$$

OBJECTIVE 1

1 Write the Augmented Matrix of a System of Linear Equations

EXAMPLE

Writing the Augmented Matrix of a System of Linear Equations

Write the augmented matrix of each system of equations.

(a) $$\begin{cases} 3x - 2y = 3 \\ -2x + y = -2 \end{cases}$$

(b) $$\begin{cases} 3x - 2y + 5 = 0 \\ -2x + 4z + 2 = 0 \\ x + 4y - 7z = 0 \end{cases}$$

OBJECTIVE 2

2 Write the System of Equations from the Augmented Matrix

EXAMPLE **Writing the System of Linear Equations from the Augmented Matrix**

Write the system of linear equations corresponding to each augmented matrix.

(a) $\left[\begin{array}{cc|c} -2 & 1 & 3 \\ 1 & 1 & -2 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 3 & -2 & 5 & 3 \\ -2 & 1 & 4 & -2 \\ 1 & 4 & -7 & 1 \end{array}\right]$

OBJECTIVE 3

3 Perform Row Operations on a Matrix

Row Operations

1. Interchange any two rows.
2. Replace a row by a nonzero multiple of that row.
3. Replace a row by the sum of that row and a constant nonzero multiple of some other row.

EXAMPLE

Applying a Row Operation to an Augmented Matrix

Apply the row operation $R_2 = 2r_1 + r_2$ to the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 3 & -4 \\ -2 & -5 & 3 \end{array}\right]$$

EXAMPLE

Finding a Particular Row Operation

Find a row operation that will result in the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 1 & -5 \end{array}\right]$$

having a 0 in row 1, column 2.

OBJECTIVE 4

4 Solve a System of Linear Equations Using Matrices

DEFINITION

A matrix is in **row echelon form** when

1. The entry in row 1, column **1** is a 1, and 0's appear below it.
2. The first nonzero entry in each row after the first row is a 1, 0's appear below it, and it appears to the right of the first nonzero entry in any row above.
3. Any rows that contain all 0's to the left of the vertical bar appear at the bottom.

EXAMPLE

How to Solve a System of Linear Equations Using Matrices

Solve: $\begin{cases} x + y - z = -1 \\ 4x - 3y + 2z = 16 \\ 2x - 2y - 3z = 5 \end{cases}$

Matrix Method for Solving a System of Linear Equations (Row Echelon Form)

STEP 1: Write the augmented matrix that represents the system.

STEP 2: Perform row operations that place the entry 1 in row 1, column 1.

STEP 3: Perform row operations that leave the entry 1 in row 1, column 1 unchanged, while causing 0's to appear below it in column 1.

STEP 4: Perform row operations that place the entry 1 in row 2, column 2, but leave the entries in columns to the left unchanged. If it is impossible to place a 1 in row 2, column 2, then proceed to place a 1 in row 2, column 3. Once a 1 is in place, perform row operations to place 0's below it.

[Place any rows that contain only 0's on the left side of the vertical bar, at the bottom of the matrix.]

STEP 5: Now repeat Step 4, placing a 1 in the next row, but one column to the right. Continue until the bottom row or the vertical bar is reached.

STEP 6: The matrix that results is the row echelon form of the augmented matrix. Analyze the system of equations corresponding to it to solve the original system.

EXAMPLE

Solving a System of Linear Equations Using Matrices (Row Echelon Form)

Solve: $\begin{cases} x + y + z = 0 \\ -2x + 3y - z = -19 \\ 4x - 3y + 4z = 28 \end{cases}$

EXAMPLE

Solving a Dependent System of Linear Equations Using Matrices

Solve: $\begin{cases} x - 3y + 2z = 6 \\ -2x + y - 3z = 10 \\ x - 8y + 3z = 28 \end{cases}$

EXAMPLE

Solving an Inconsistent System of Linear Equations Using Matrices

Solve: $\begin{cases} 3x - 4y + 8z = 4 \\ 9x + 13y + 49z = 0 \\ -3x + 3y - 9z = 1 \end{cases}$

EXAMPLE

Solving a System of Linear Equations Using Matrices

Solve:

$$\begin{cases} x - 2y + z = 0 \\ 2x + 2y - 3z = -3 \\ y - z = -1 \\ -x + 4y + 2z = 13 \end{cases}$$

EXAMPLE Penalties in the 2006 Fifa World Cup

Italy and France combined for a total of 46 penalties during the 2006 Fifa World Cup. The penalties were a combination of fouls, yellow cards (cautions), and red cards (expulsions). There was one less red card than half the number of yellow cards and one more foul than 8 times the total number of cards. How many of each type of penalty were there during the match?