Section 3.1

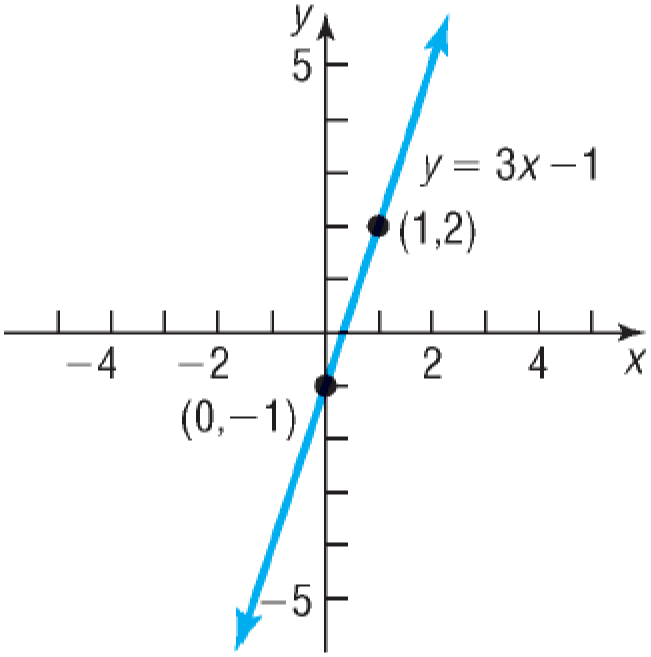
Functions

OBJECTIVE 1

1 Determine Whether a Relation Represents a Function

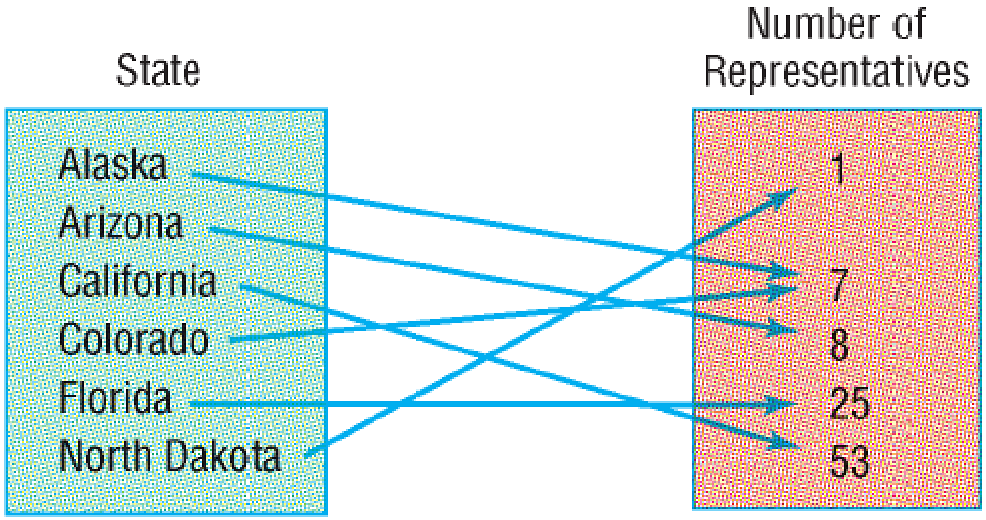
A **relation** is a correspondence between two sets.

If x and y are two elements in these sets and if a relation exists between x and y, then we say that x corresponds to y or that y depends on x, and we write $x \rightarrow y$.

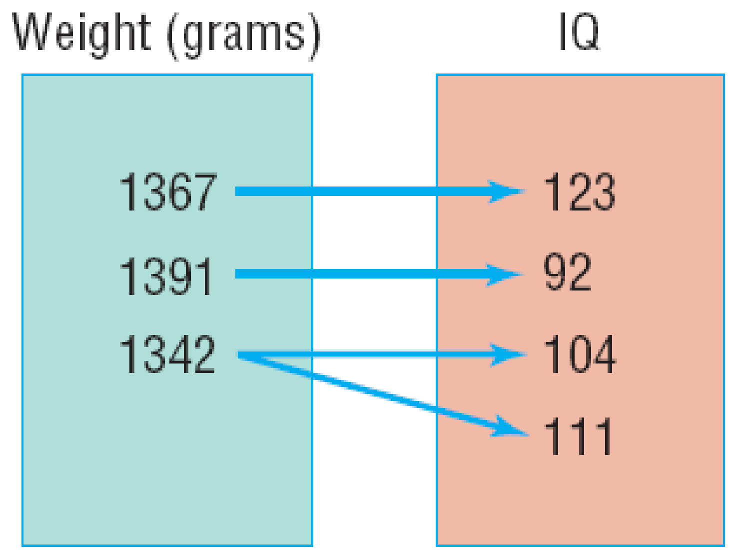


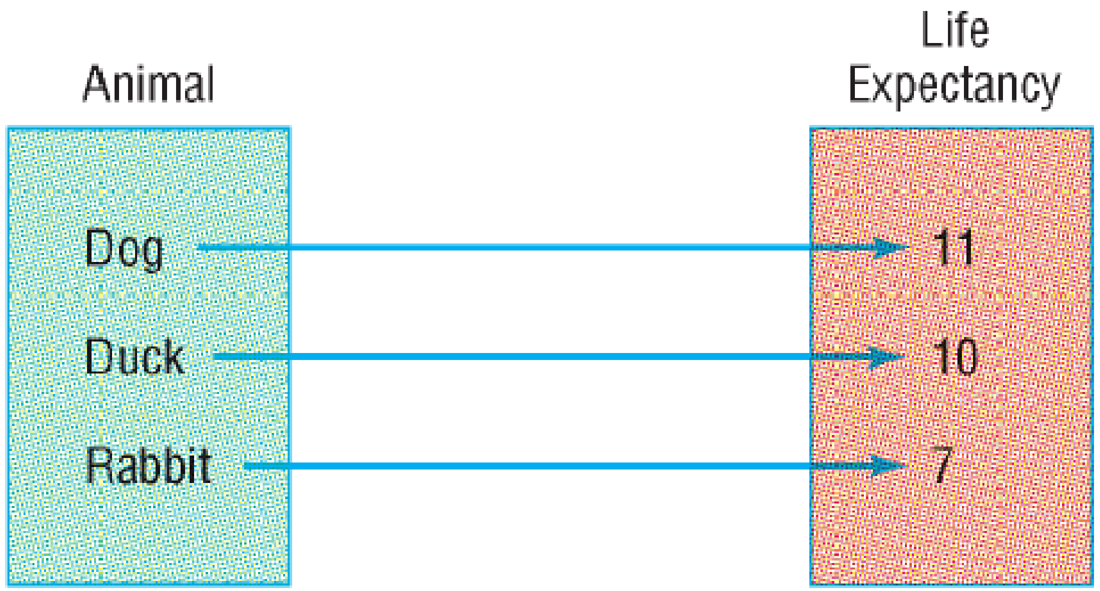
EXAMPLE

Maps and Ordered Pairs as Relations



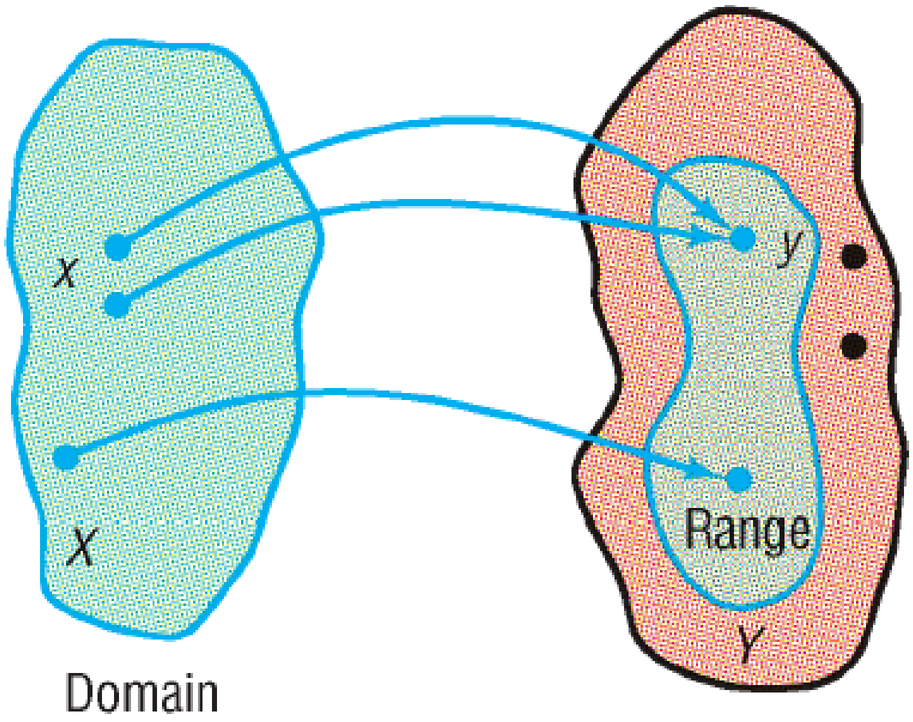
{(Alaska, 7), (Arizona, 8), (California, 53),
(Colorado, 7), (Florida, 25), (North Dakota, 1)}


Weight (grams)
IQ
1367
1391
1342
123
92
104
111


Animal
Life Expectancy
Dog
Duck
Rabbit
11
10
7

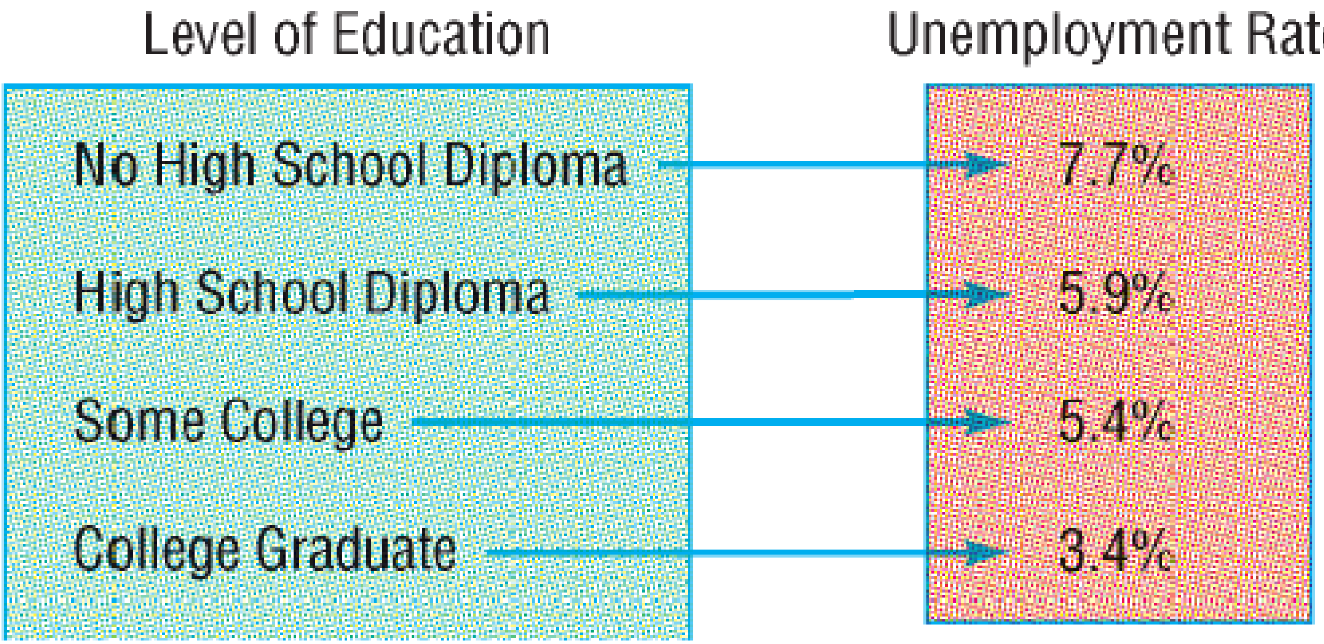
FUNCTION

Let X and Y be two nonempty sets.* A **function** from X into Y is a relation that associates with each element of X exactly one element of Y.



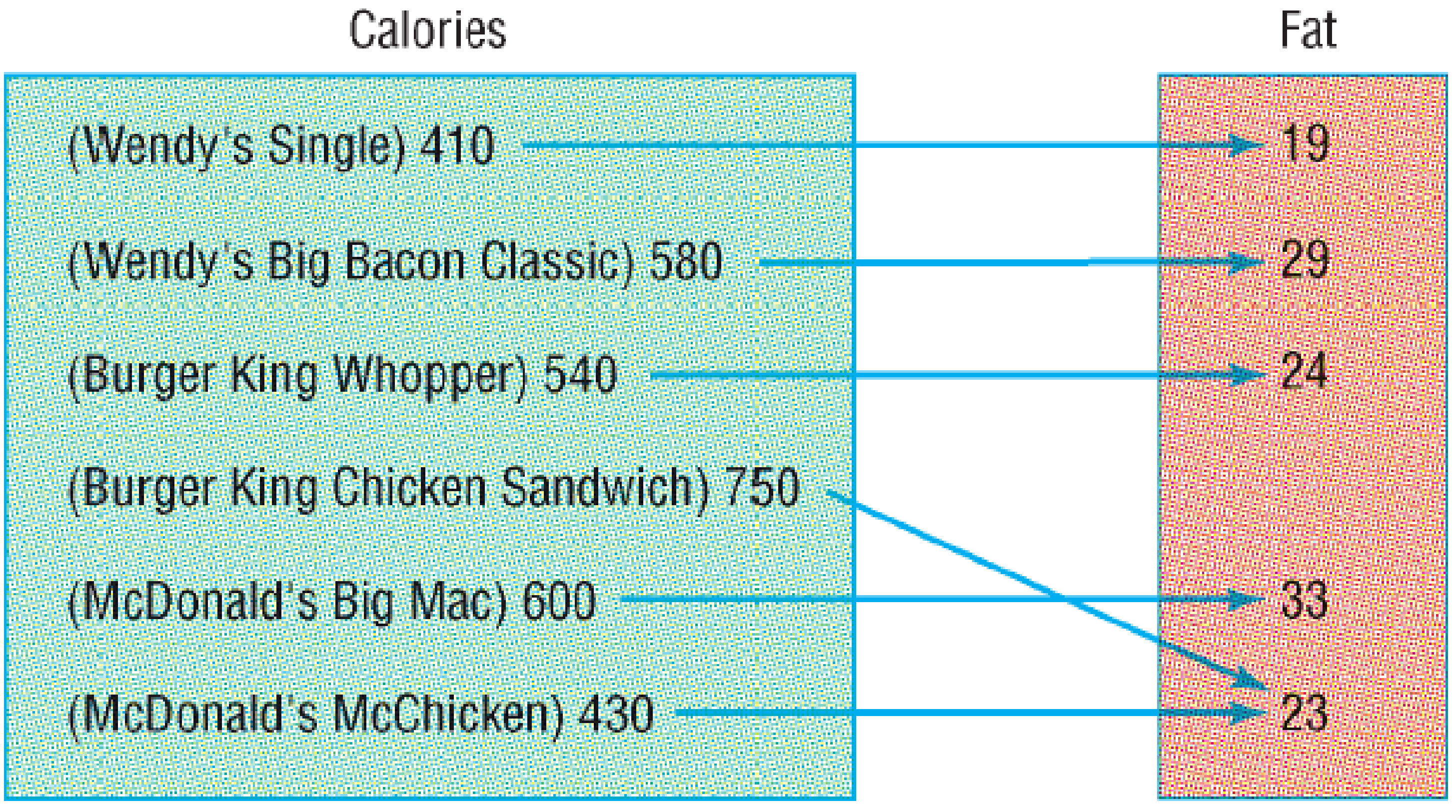
EXAMPLE

Determining Whether a Relation Represents a Function



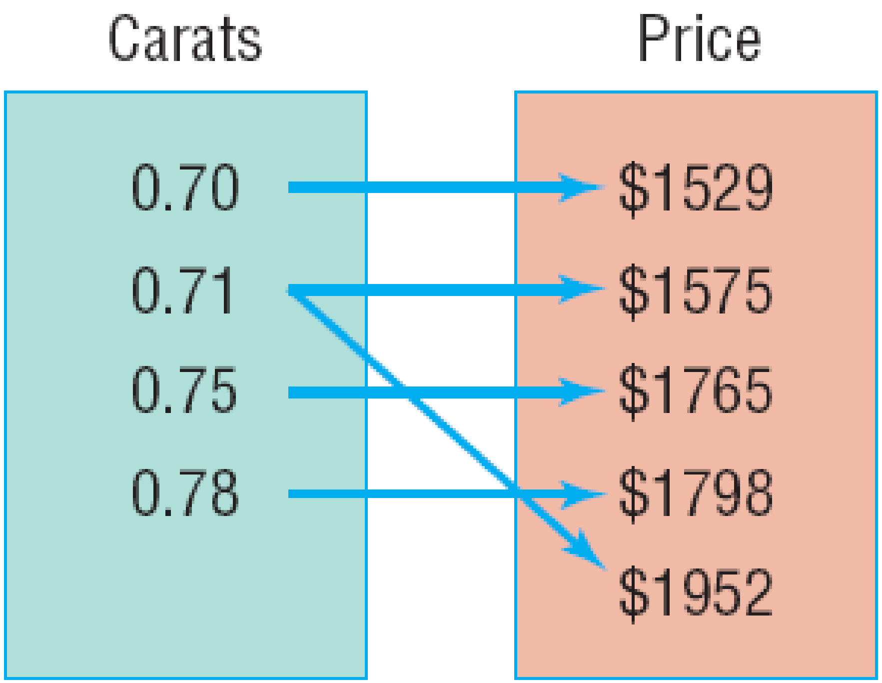
EXAMPLE

Determining Whether a Relation Represents a Function



EXAMPLE

Determining Whether a Relation Represents a Function



EXAMPLE

Determining Whether a Relation Represents a Function

Determine whether each relation represents a function. If it is a function, state the domain and range.

{(2, 3), (4, 1), (3, -2), (2, -1)}

{(-2, 3), (4, 1), (3, -2), (2, -1)}

{(2, 3), (4, 3), (3, 3), (2, -1)}

EXAMPLE

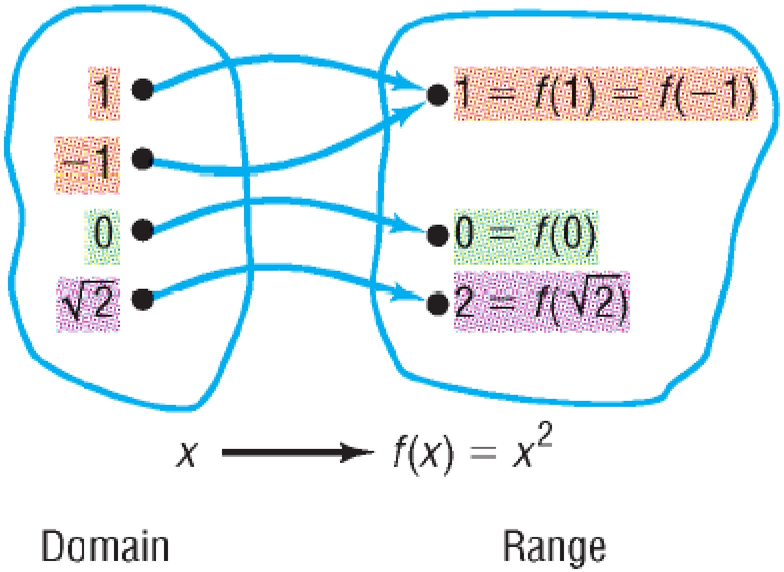
Determining Whether an Equation Is a Function

Determine if the equation $y = -\frac{1}{2}x - 3$ defines *y* as a function of *x*.

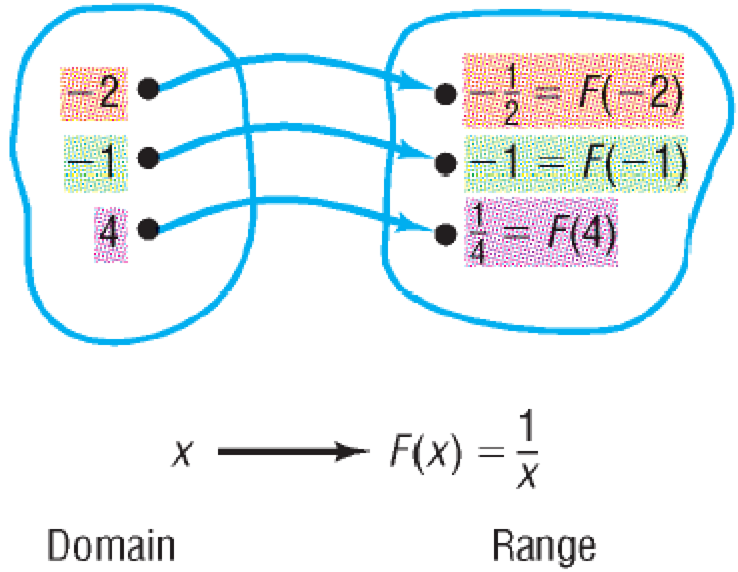
Determine if the equation $x = 2y^2 + 1$ defines *y* as a function of *x*.

OBJECTIVE 2

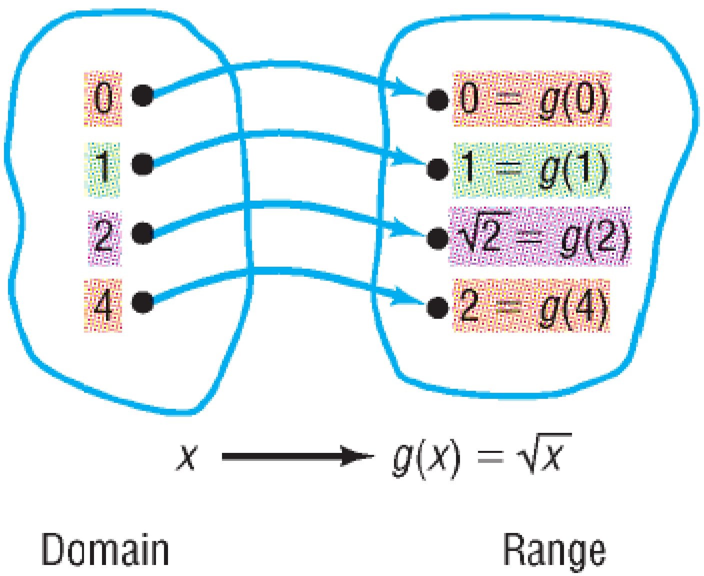
2 Find the Value of a Function



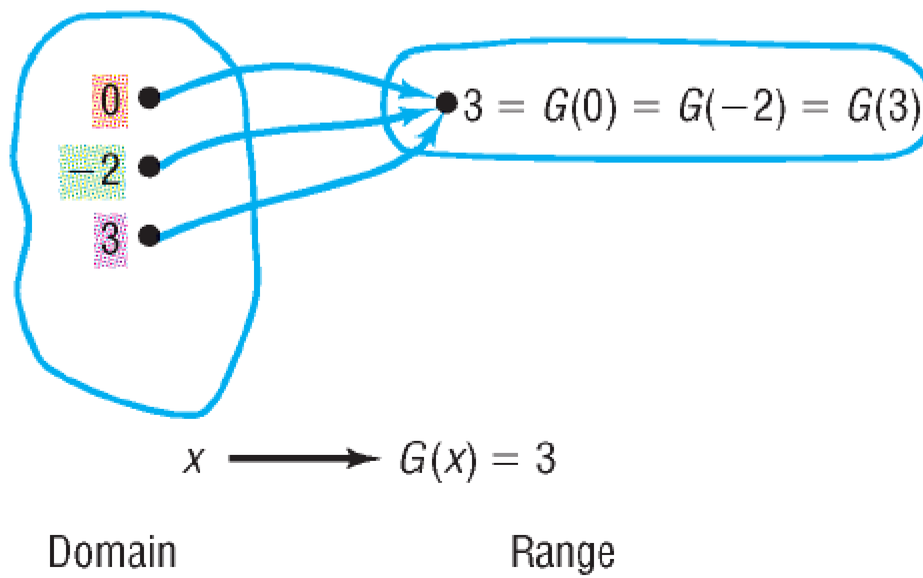
(a) $f(x) = x^2$



(b) $F(x) = \frac{1}{x}$

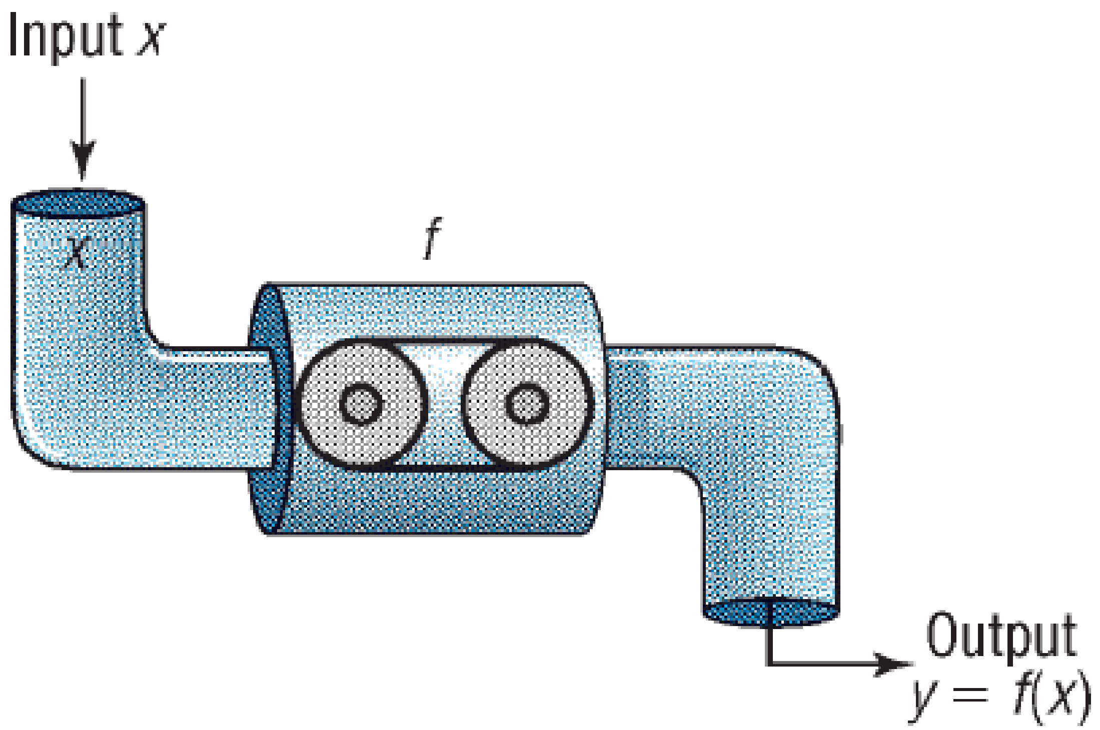


(c) $g(x) = \sqrt{x}$



(d) $G(x) = 3$

FUNCTION MACHINE



1. It only accepts numbers from the domain of the function.

2. For each input, there is exactly one output (which may be repeated for different inputs).

EXAMPLE

Illustrating Language Used with Functions

The name of the function is *f*.

The *independent variable* or *argument* of *f* is *p*.

$$f(p) = p^2$$

$$\text{If } q = f(p)$$

q is called the *dependent variable*

$$f(3) = 9$$

9 is the *value* of *f* at 3 or 9 is the *image* of 3.

State the *domain* and the *range* of *f*.

EXAMPLE

Finding Values of a Function

For the function f defined by $f(x) = -3x^2 + 2x$, evaluate:

(a) $f(3)$

(b) $f(x) + f(3)$

(c) $3f(x)$

(d) $f(-x)$

(e) $-f(x)$

(f) $f(3x)$

(g) $f(x + 3)$

(h) $\dfrac{f(x + h) - f(x)}{h} \quad h \neq 0$

EXAMPLE

Finding Values of a Function on a Calculator

(a) $f(x) = x^2; \quad f(1.234) =$

(b) $F(x) = \dfrac{1}{x}; \quad F(1.234) =$

(c) $g(x) = \sqrt{x}; \quad g(1.234) =$

Implicit Form of a Function

Implicit Form	Explicit Form
$3x + y = 5$	$y = f(x) = -3x + 5$
$x^2 - y = 6$	$y = f(x) = x^2 - 6$
$xy = 4$	$y = f(x) = \dfrac{4}{x}$

Summary

Important Facts About Functions

(a) For each x in the domain of f, there is exactly one image $f(x)$ in the range; however, an element in the range can result from more than one x in the domain.

(b) f is the symbol that we use to denote the function. It is symbolic of the equation that we use to get from an x in the domain to $f(x)$ in the range.

(c) If $y = f(x)$, then x is called the independent variable or argument of f, and y is called the dependent variable or the value of f at x.

OBJECTIVE 3

3 Find the Domain of a Function Defined by an Equation

EXAMPLE Finding the Domain of a Function

Find the domain of each of the following functions:

(a) $f(x) = \dfrac{x+4}{x^2 - 2x - 3}$

(b) $g(x) = x^2 - 9$

(c) $h(x) = \sqrt{3 - 2x}$

Finding the Domain of a Function Defined by an Equation

1. Start with the domain as the set of real numbers.
2. If the equation has a denominator, exclude any numbers that give a zero denominator.
3. If the equation has a radical of even index, exclude any numbers that cause the expression inside the radical to be negative.

EXAMPLE

Finding the Domain in an Application

A rectangular garden has a perimeter of 100 feet. Express the area *A* of the garden as a function of the width *w*. Find the domain.

OBJECTIVE 4

4 **Form the Sum, Difference, Product, and Quotient of Two Functions**

If f and g are functions:

The **sum** $\boldsymbol{f + g}$ is the function defined by

$$(f+g)(x) = f(x) + g(x)$$

The **difference** $\boldsymbol{f - g}$ is the function defined by

$$(f-g)(x) = f(x) - g(x)$$

The product $\boldsymbol{f \cdot g}$ is the function defined by

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

The **quotient** $\boldsymbol{\dfrac{f}{g}}$ is the function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0$$

EXAMPLE **Operations on Functions**

For the functions $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$ find the following:

(a) $(f + g)(x)$

(b) $(f - g)(x)$

(c) $(f \cdot g)(x)$

(d) $\left(\dfrac{f}{g}\right)(x)$

Summary

Function

A relation between two sets of real numbers so that each number x in the first set, the domain, has corresponding to it exactly one number y in the second set.
A set of ordered pairs (x, y) or $(x, f(x))$ in which no first element is paired with two different second elements.
The range is the set of y values of the function for the x values in the domain.
A function f may be defined implicitly by an equation involving x and y or explicitly by writing $y = f(x)$.

Unspecified domain

If a function f is defined by an equation and no domain is specified, then the domain will be taken to be the largest set of real numbers for which the equation defines a real number.

Function notation

$y = f(x)$
f is a symbol for the function.
x is the independent variable or argument.
y is the dependent variable.
$f(x)$ is the value of the function at x, or the image of x.