OBJECTIVE 1

Determine Even and Odd Functions from a Graph
A function $f$ is **even** if, for every number $x$ in its domain, the number $-x$ is also in the domain and

$$f(-x) = f(x)$$

For an **even** function, for every point $(x, y)$ on the graph, the point $(-x, y)$ is also on the graph.
A function $f$ is **odd** if, for every number $x$ in its domain, the number $-x$ is also in the domain and

$$f(-x) = -f(x)$$

For an **odd** function, for every point $(x, y)$ on the graph, the point $(-x, -y)$ is also on the graph.
Theorem

A function is even if and only if its graph is symmetric with respect to the y-axis.
A function is odd if and only if its graph is symmetric with respect to the origin.
Determine whether each graph given is an even function, an odd function, or a function that is neither even nor odd.
OBJECTIVE 2

Identify Even and Odd Functions from the Equation
Identifying Even and Odd Functions

Use a graphing utility to conjecture whether each of the following functions is even, odd, or neither. Verify the conjecture algebraically. Then state whether the graph is symmetric with respect to the y-axis or with respect to the origin.

\[ f(x) = -3x^4 - x^2 + 2 \]

\[ g(x) = 5x^3 - 1 \]

\[ h(x) = 2x^3 - x \]
OBJECTIVE 3

Use a Graph to Determine Where a Function Is Increasing, Decreasing, or Constant
Where is the function increasing?
Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function decreasing?
EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function constant?
A function $f$ is **increasing** on an open interval $I$ if, for any choice of $x_1$ and $x_2$ in $I$, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

A function $f$ is **decreasing** on an open interval $I$ if, for any choice of $x_1$ and $x_2$ in $I$, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

A function $f$ is **constant** on an interval $I$ if, for all choices of $x$ in $I$, the values $f(x)$ are equal.
(a) For $x_1 < x_2$ in $I$, $f(x_1) < f(x_2)$; $f$ is increasing on $I$

(b) For $x_1 < x_2$ in $I$, $f(x_1) > f(x_2)$; $f$ is decreasing on $I$

(c) For all $x$ in $I$, the values of $f$ are equal; $f$ is constant on $I$
OBJECTIVE 4

Use a Graph to Locate Local Maxima and Local Minima
The local maximum is $f(c)$ and occurs at $x = c$. Increasing $ightarrow$ decreasing.
The local minimum is $f(c)$ and occurs at $x = c$. 

Viewing the relationship between $x$ and $f(x)$, we see that as $x$ moves from $a$ to $c$, $f(x)$ is decreasing. But as $x$ moves from $c$ to $b$, $f(x)$ is increasing.
A function $f$ has a **local maximum** at $c$ if there is an open interval $I$ containing $c$ so that, for all $x \neq c$ in $I$, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum of $f$**.

A function $f$ has a **local minimum** at $c$ if there is an open interval $I$ containing $c$ so that, for all $x \neq c$ in $I$, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum of $f$**.
Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant

(a) At what number(s), if any, does $f$ have a local maximum?
(b) What are the local maxima?
Example

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant

(c) At what number(s), if any, does $f$ have a local minimum?

(d) What are the local minima?
(e) List the intervals on which \( f \) is increasing. List the intervals on which \( f \) is decreasing.
OBJECTIVE 5

Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing
EXAMPLE

Using a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing

Use a graphing utility to graph \( f(x) = 2x^3 - 3x + 1 \) for \(-2 < x < 2\).
Approximate where \( f \) has any local maxima or local minima.
OBJECTIVE 6

Find the Average Rate of Change of a Function
If $a$ and $b$, $a \neq b$, are in the domain of a function $y = f(x)$, the **average rate of change** of $f$ from $a$ to $b$ is defined as

\[
\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} \quad a \neq b \tag{1}
\]
EXAMPLE

Finding the Average Rate of Change

Find the average rate of change of \( f(x) = \frac{1}{2}x^2 \):

From 0 to 1 \hspace{1cm} From 0 to 3 \hspace{1cm} From 0 to 5

\[
\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}
\]
The Secant Line

\[ m_{\text{sec}} = \frac{f(b) - f(a)}{b - a} \]
Theorem

Slope of the Secant Line

The average rate of change of a function from $a$ to $b$ equals the slope of the secant line containing two points $(a, f(a))$ and $(b, f(b))$ on its graph.
EXAMPLE

Finding the Equation of a Secant Line

Suppose that $g(x) = -2x^2 + 4x - 3$.

(a) Find the average rate of change of $g$ from $-2$ to $1$.
(b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.
(c) Using a graphing utility, draw the graph of $g$ and the secant line obtained in part (b) on the same screen.