

Section 3.4

Library of Functions;

Piecewise-defined Functions

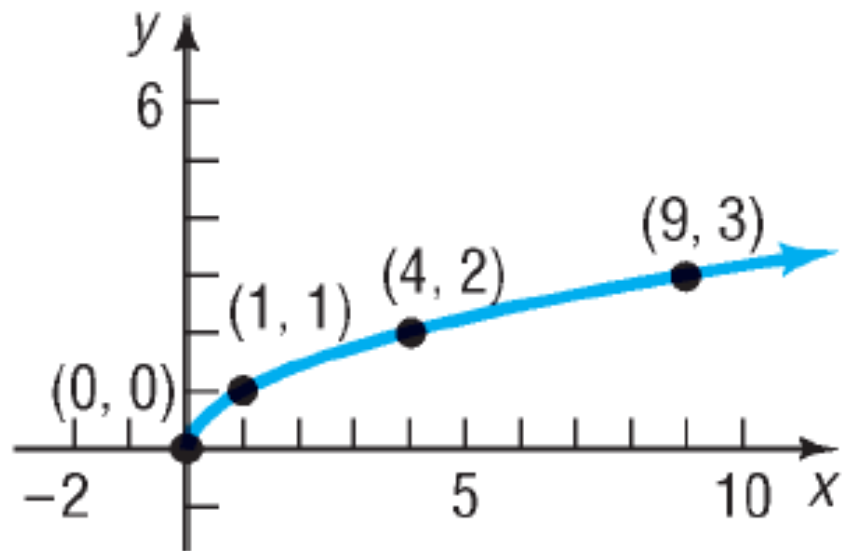
OBJECTIVE 1

 **Graph the Functions Listed in the Library of Functions**

The Square Root Function

Properties of $f(x) = \sqrt{x}$

1. The domain and the range are the set of nonnegative real numbers.
2. The x -intercept of the graph of $f(x) = \sqrt{x}$ is 0. The y -intercept of the graph of $f(x) = \sqrt{x}$ is also 0.
3. The function is neither even nor odd.
4. It is increasing on the interval $(0, \infty)$.
5. It has a minimum value of 0 at $x = 0$.

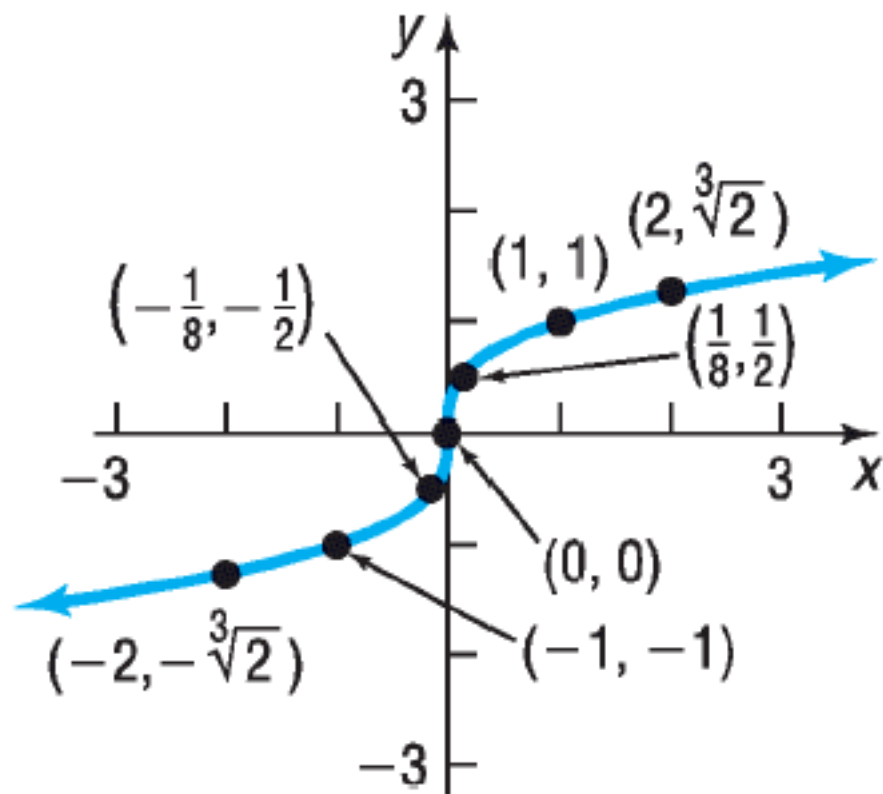


EXAMPLE

Graphing the Cube Root Function

- (a) Determine whether $f(x) = \sqrt[3]{x}$ is even, odd, or neither. State whether the graph of f is symmetric with respect to the y -axis or symmetric with respect to the origin.
- (b) Determine the intercepts, if any, of the graph of $f(x) = \sqrt[3]{x}$.
- (c) Graph $f(x) = \sqrt[3]{x}$.

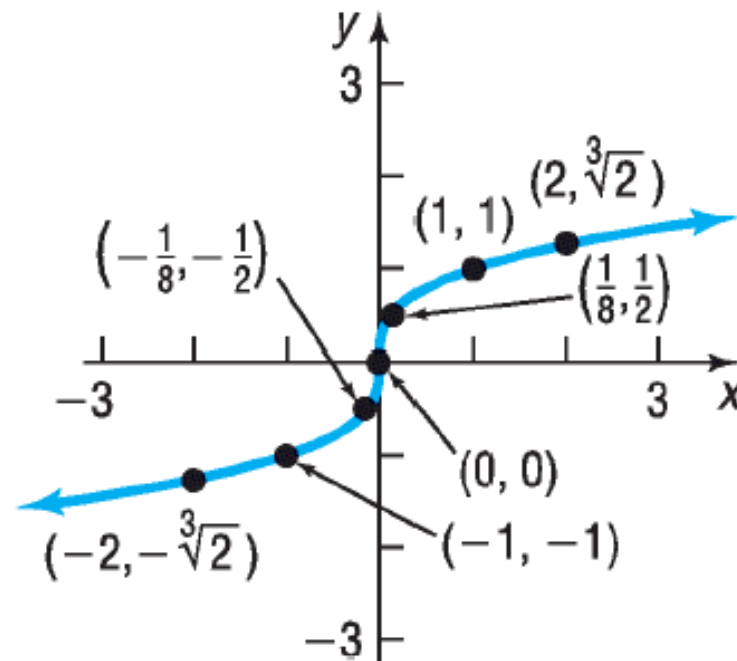
x	$y = f(x) = \sqrt[3]{x}$	(x, y)
0	0	$(0, 0)$
$\frac{1}{8}$	$\frac{1}{2}$	$(\frac{1}{8}, \frac{1}{2})$
1	1	$(1, 1)$
2	$\sqrt[3]{2} \approx 1.26$	$(2, \sqrt[3]{2})$
8	2	$(8, 2)$



The Cube Root Function

Properties of $f(x) = \sqrt[3]{x}$

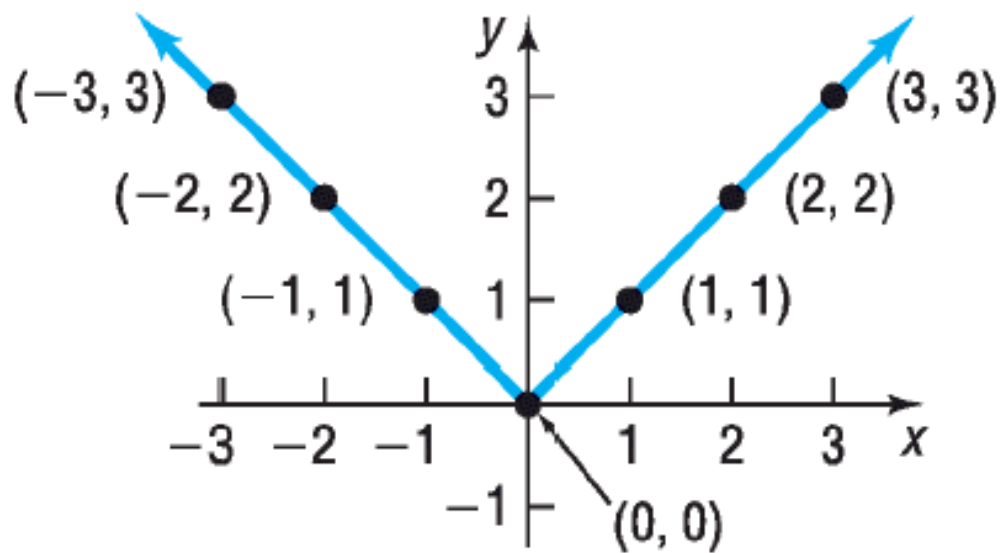
1. The domain and the range are the set of all real numbers.
2. The x -intercept of the graph of $f(x) = \sqrt[3]{x}$ is 0. The y -intercept of the graph of $f(x) = \sqrt[3]{x}$ is also 0.
3. The graph is symmetric with respect to the origin. The function is odd.
4. It is increasing on the interval $(-\infty, \infty)$.
5. It does not have a local minimum or a local maximum.



EXAMPLE Graphing the Absolute Value Function

- (a) Determine whether $f(x) = |x|$ is even, odd, or neither. State whether the graph of f is symmetric with respect to the y -axis or symmetric with respect to the origin.
- (b) Determine the intercepts, if any, of the graph of $f(x) = |x|$.
- (c) Graph $f(x) = |x|$.

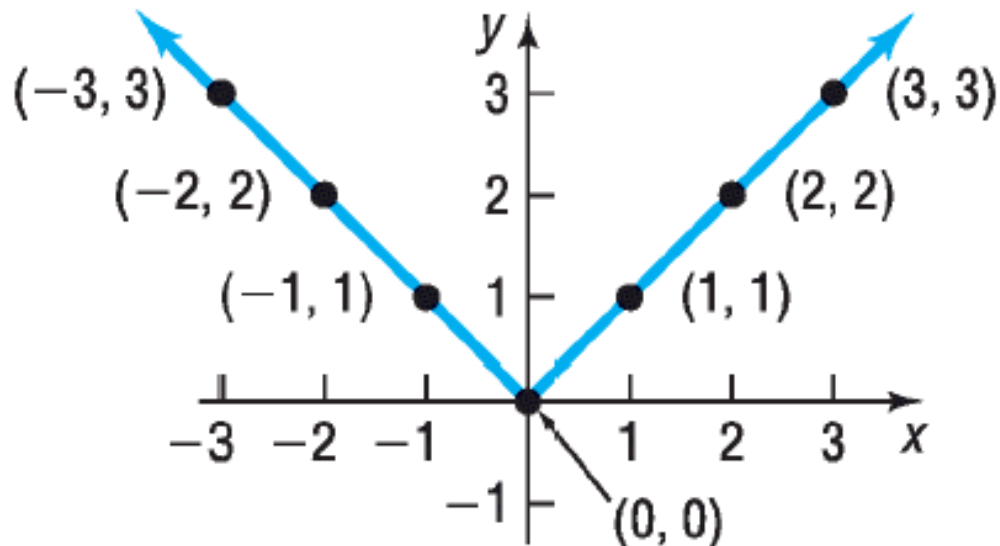
x	$y = f(x) = x $	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)



The Absolute Value Function

Properties of $f(x) = |x|$

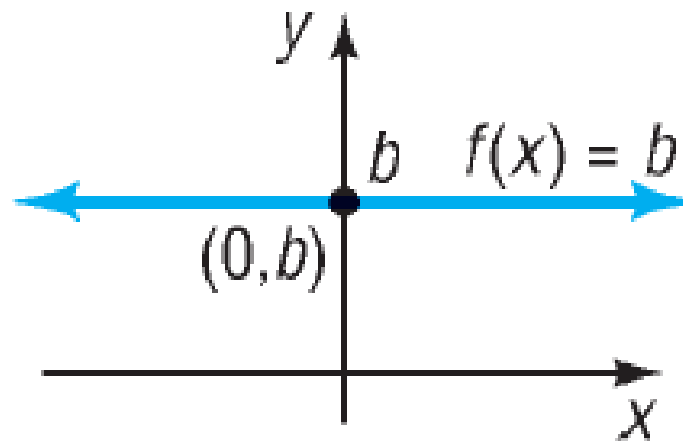
1. The domain is the set of all real numbers. The range of f is $\{y|y \geq 0\}$.
2. The x -intercept of the graph of $f(x) = |x|$ is 0. The y -intercept of the graph of $f(x) = |x|$ is also 0.
3. The graph is symmetric with respect to the y -axis. The function is even.
4. It is decreasing on the interval $(-\infty, 0)$. It is increasing on the interval $(0, \infty)$.
5. It has a local minimum of 0 at $x = 0$.



Constant Function

$$f(x) = b, \quad b \text{ is a real number}$$

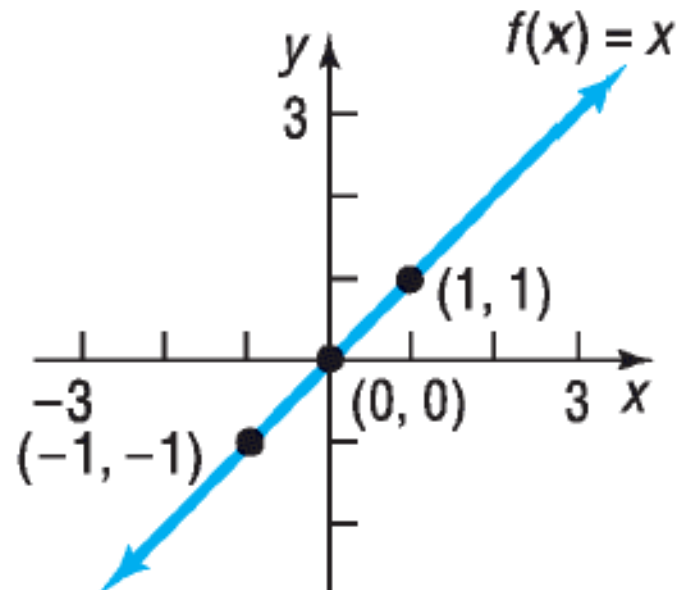
Constant Function



Identity Function

$$f(x) = x$$

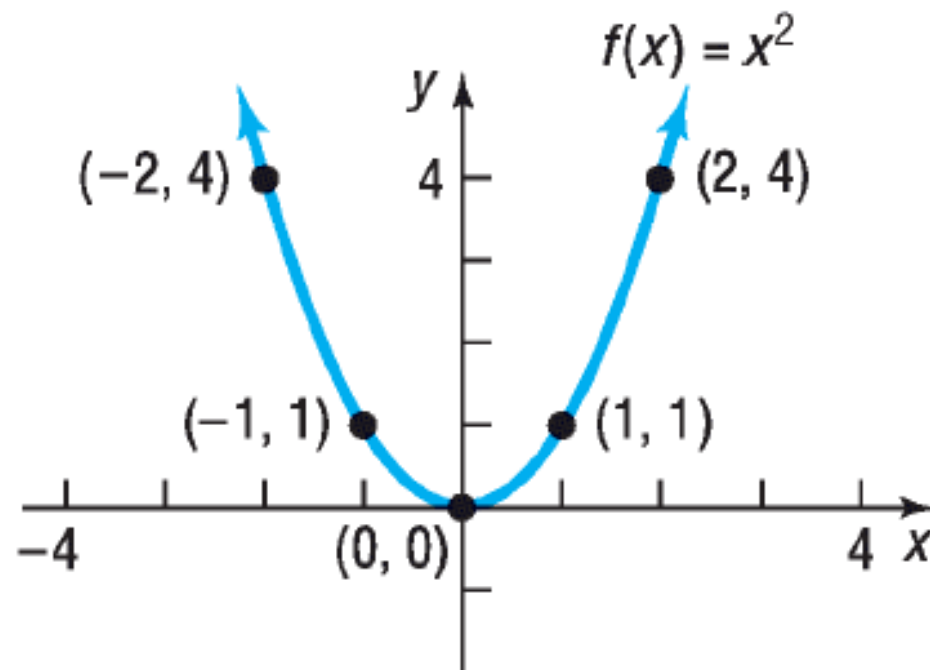
Identity Function



Square Function

$$f(x) = x^2$$

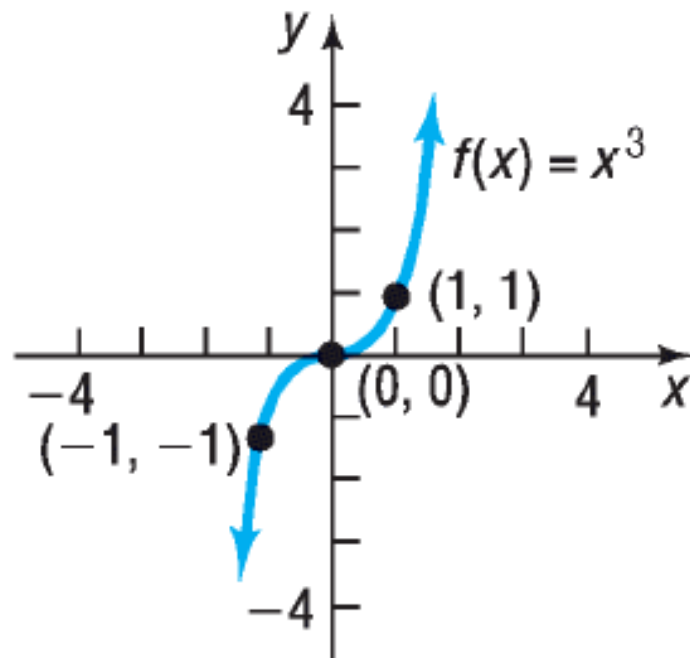
Square Function



Cube Function

$$f(x) = x^3$$

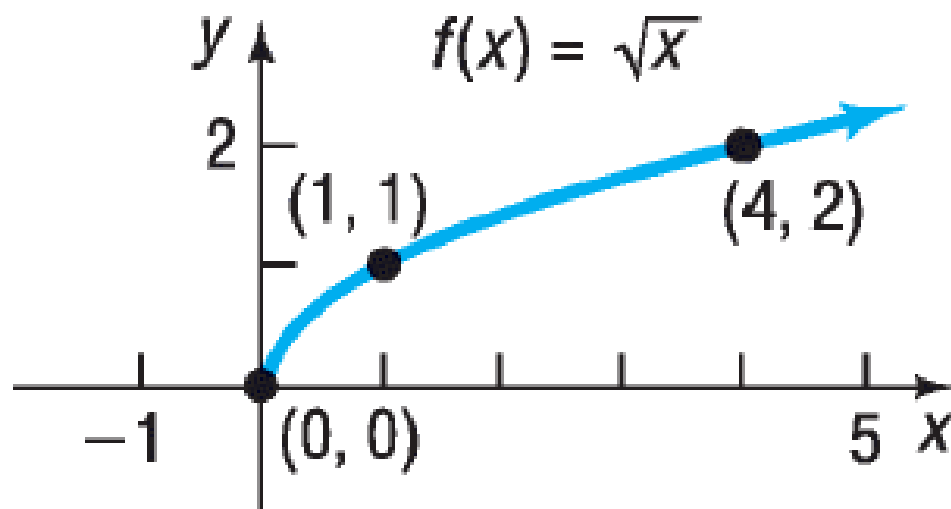
Cube Function



Square Root Function

$$f(x) = \sqrt{x}$$

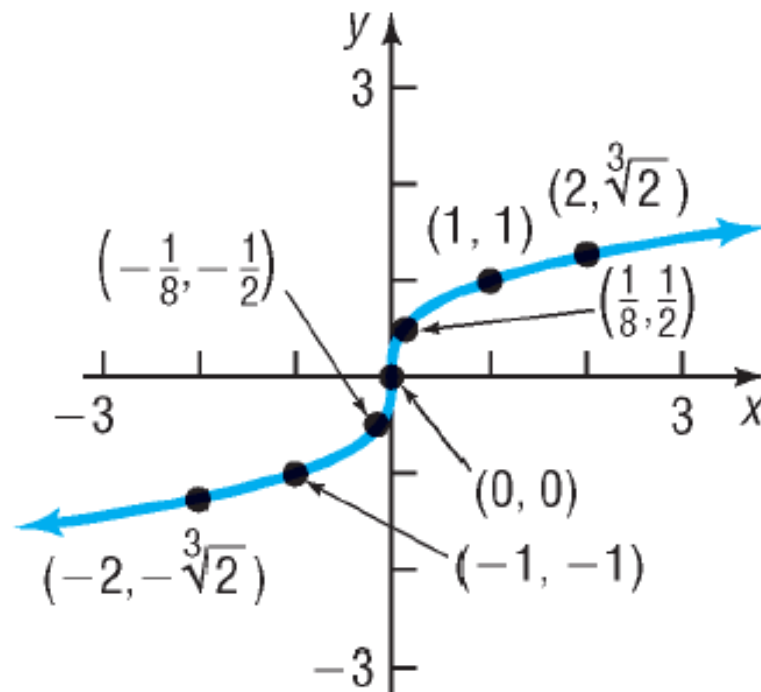
Square Root Function



Cube Root Function

$$f(x) = \sqrt[3]{x}$$

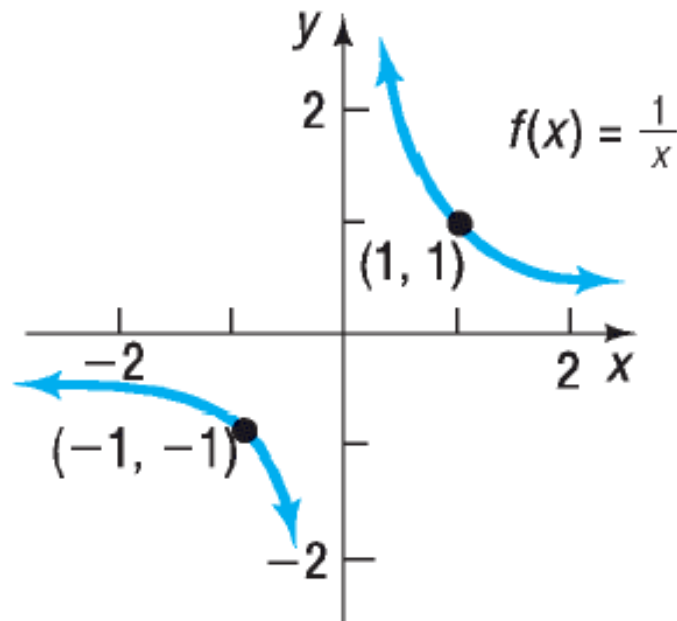
Cube Root Function



Reciprocal Function

$$f(x) = \frac{1}{x}$$

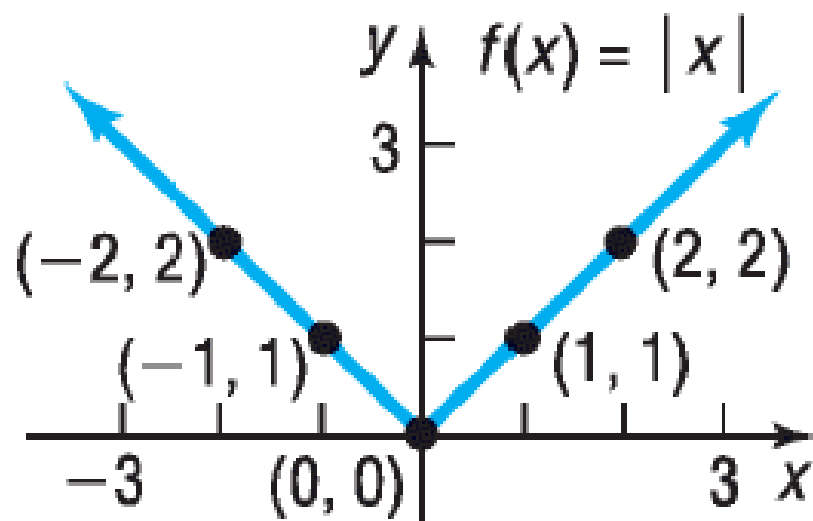
Reciprocal Function



Absolute Value Function

$$f(x) = |x|$$

Absolute Value Function

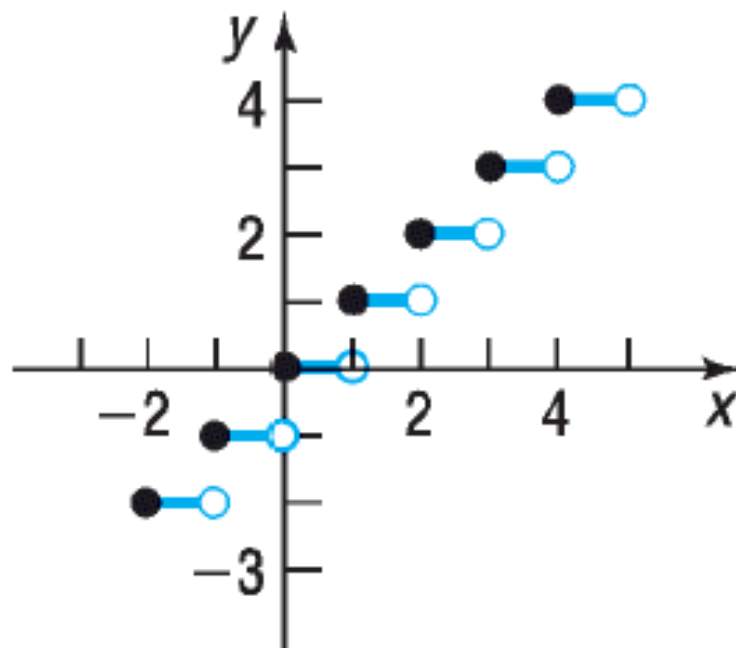


Greatest Integer Function

$f(x) = \text{int}(x)^* =$ greatest integer less than or equal to x

x	$y = f(x)$ $= \text{int}(x)$	(x, y)
-1	-1	$(-1, -1)$
$-\frac{1}{2}$	-1	$(-\frac{1}{2}, -1)$
$-\frac{1}{4}$	-1	$(-\frac{1}{4}, -1)$
0	0	$(0, 0)$
$\frac{1}{4}$	0	$(\frac{1}{4}, 0)$
$\frac{1}{2}$	0	$(\frac{1}{2}, 0)$
$\frac{3}{4}$	0	$(\frac{3}{4}, 0)$

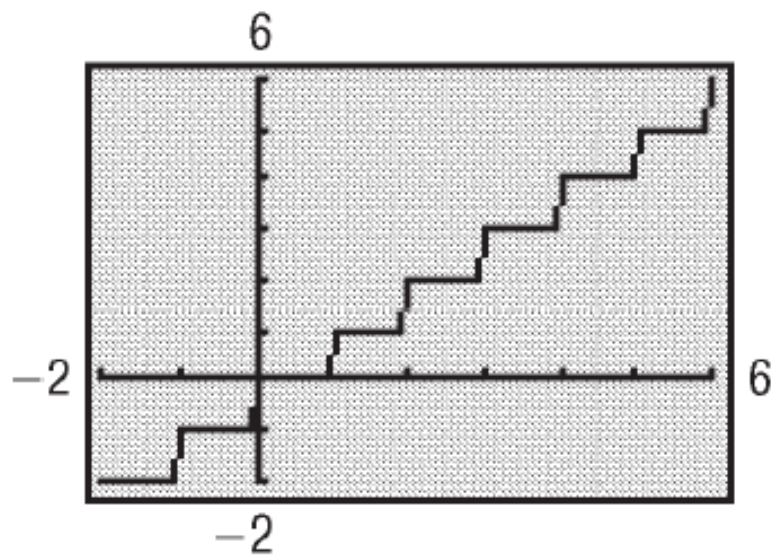
Greatest Integer Function



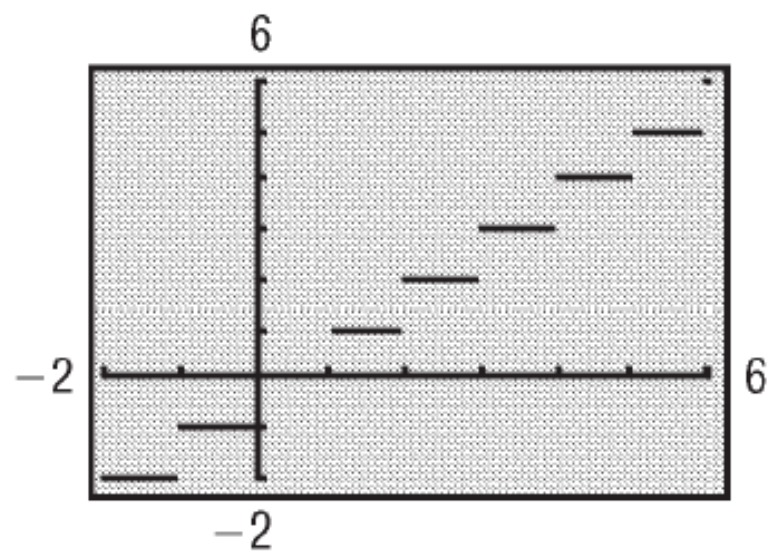
Greatest Integer Function

$f(x) = \text{int}(x)^* = \text{greatest integer less than or equal to } x$

$$f(x) = \text{int}(x)$$



(a) Connected mode



(b) Dot mode

OBJECTIVE 2



Graph Piecewise-defined Functions

EXAMPLE

Analyzing a Piecewise-defined Function

The function f is defined as

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ x + 2 & \text{if } x > 0 \end{cases}$$

(a) Find $f(-2)$, $f(0)$, and $f(3)$.

(b) Determine the domain of f .

(c) Graph f .

(d) Use the graph to find the range of f .

(e) Is f continuous on its domain?

EXAMPLE**Cost of Electricity**

In May 2006, Commonwealth Edison Company supplied electricity to residences for a monthly customer charge of \$7.58 plus 8.275¢ per kilowatt-hour (kWhr) for the first 400 kWhr supplied in the month and 6.208¢ per kWhr for all usage over 400 kWhr in the month.

- (a) What is the charge for using 300 kWhr in a month?
- (b) What is the charge for using 700 kWhr in a month?
- (c) If C is the monthly charge for x kWhr, develop a model relating the monthly charge and kilowatt-hours used. That is, express C as a function of x .