Section 3.6

Mathematical Models: Building Functions

OBJECTIVE 1

1 Build and Analyze Functions

EXAMPLE

Finding the Distance from the Origin to a Point on a Graph

Let $P = (x, y)$ be a point on the graph of $y = x^2 - 4$

(a) Express the distance d from P to the origin O as a function of x.

(b) What is d if $x = 0$?

(c) What is d if $x = 1$?

(d) What is d if $x = \dfrac{\sqrt{2}}{2}$?

(e) Use a graphing utility to graph the function $d = d(x)$, $x \geq 0$. Rounded to two decimal places, find the value(s) of x at which d has a local minimum.

EXAMPLE Area of a Rectangle

A rectangle has one corner in quadrant I on the graph of $y = 9 - x^2$ another at the origin, a third on the positive y-axis, and the fourth on the positive x-axis.

(a) Express the area A of the rectangle as a function of x.

(b) What is the domain of A?

(c) Graph $A = A(x)$.

(d) For what value of x is the area largest?

EXAMPLE Making a Playpen*

A manufacturer of children's playpens makes a square model that can be opened at one corner and attached at right angles to a wall or, perhaps, the side of a house. If each side is 3 feet in length, the open configuration doubles the available area in which the child can play from 9 square feet to 18 square feet. See Figure 85.

Now suppose that we place hinges at the outer corners to allow for a configuration like the one shown in Figure 86.

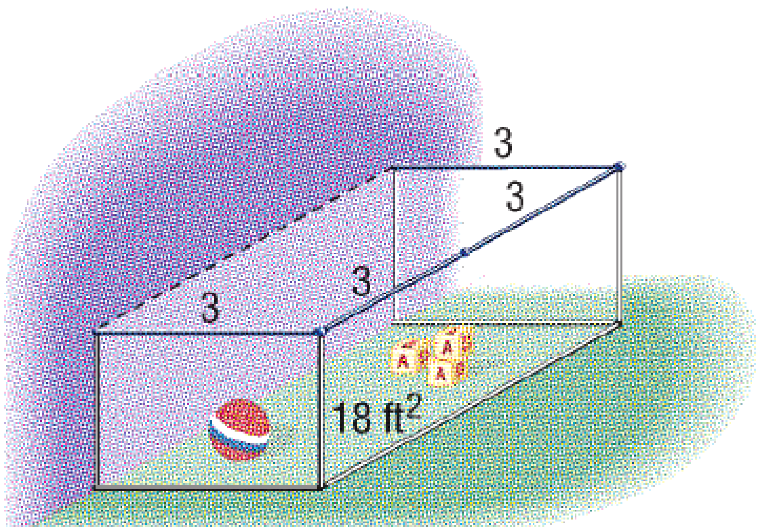

Figure 85

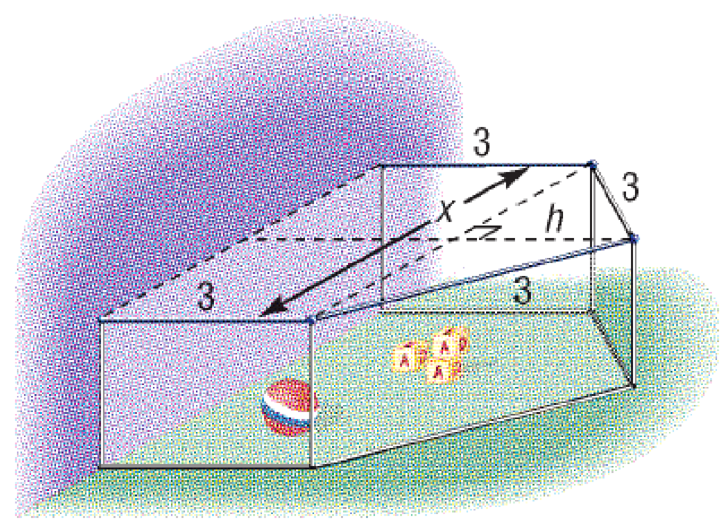

Figure 86

EXAMPLE Making a Playpen*

(a) Express the area A of this configuration as a function of the distance x between the two parallel sides.

(b) Find the domain of A.

(c) Find A if $x = 5$.

(d) Graph $A = A(x)$. For what value of x is the area largest? What is the maximum area?

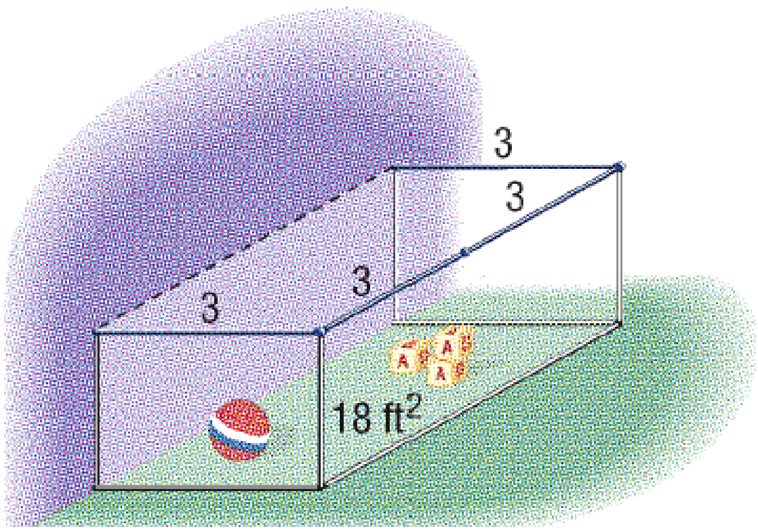

Figure 85

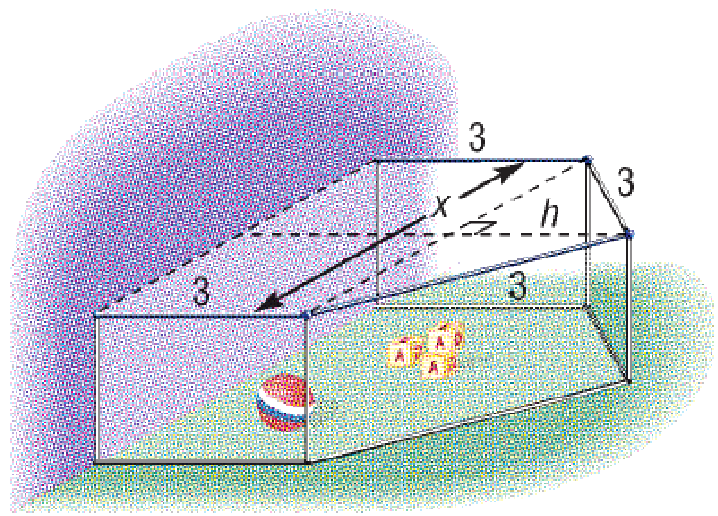

Figure 86