Section 4.1

Linear Functions, Their Properties, and Linear Models

OBJECTIVE 1

Graph Linear Functions

A **linear function** is a function of the form

$$f(x) = mx + b$$

The graph of a linear function is a line with slope m and y-intercept b. Its domain is the set of all real numbers. If $m \neq 0$, the range of a linear function is the set of all real numbers. If $m = 0$, the range is $\{y | y = b\}$.

EXAMPLE Graphing a Linear Function

Graph the linear function $f(x) = -\frac{3}{2}x + 5$

What is the domain and the range of *f*?

OBJECTIVE 2

2 Use Average Rate of Change to Identify Linear Functions

x	$y = f(x) = -3x + 7$	Average Rate of Change $= \dfrac{\Delta y}{\Delta x}$
-2	13	
		$\dfrac{10 - 13}{-1 - (-2)} = \dfrac{-3}{1} = -3$
-1	10	
		$\dfrac{7 - 10}{0 - (-1)} = \dfrac{-3}{1} = -3$
0	7	
		-3
1	4	
		-3
2	1	
		-3
3	-2	

Theorem

Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is

$$\frac{\Delta y}{\Delta x} = m$$

EXAMPLE

Using the Average Rate of Change to Identify Linear Functions

A strain of E-coli Beu 397-recA441 is placed into a Petri dish at 30° Celsius and allowed to grow. The data shown in Table 2 are collected. The population is measured in grams and the time in hours. Plot the ordered pairs (x, y) in the Cartesian plane and use the average rate of change to determine whether the function is linear.

Time (hours), x	Population (grams), y	(x, y)
0	0.09	(0, 0.09)
1	0.12	(1, 0.12)
2	0.16	(2, 0.16)
3	0.22	(3, 0.22)
4	0.29	(4, 0.29)
5	0.39	(5, 0.39)

EXAMPLE Using the Average Rate of Change to Identify Linear Functions

The data in Table 3 represent the maximum number of heartbeats that a healthy individual should have during a 15-second interval of time while exercising for different ages. Plot the ordered pairs (x, y) in the Cartesian plane and use the average rate of change to determine whether the function is linear.

Age, x	Maximum Number of Heart Beats, y	(x, y)
20	50	(20, 50)
30	47.5	(30, 47.5)
40	45	(40, 45)
50	42.5	(50, 42.5)
60	40	(60, 40)
70	37.5	(70, 37.5)

OBJECTIVE 3

3 Determine Whether a Linear Function Is Increasing, Decreasing, or Constant

Theorem

Increasing, Decreasing, and Constant Linear Functions

A linear function $f(x) = mx + b$ is increasing over its domain if its slope, m, is positive. It is decreasing over its domain if its slope, m, is negative. It is constant over its domain if its slope, m, is zero.

EXAMPLE

Determining Whether a Linear Function Is Increasing, Decreasing, or Constant

Determine whether the following linear functions are increasing, decreasing, or constant.

(a) $f(x) = -\frac{1}{2}x + 5$

(b) $g(x) = -1$

(c) $h(x) = \frac{1}{4}x - 3$

(d) $s(t) = -x + 2$

OBJECTIVE 4

4 Build Linear Models from Verbal Descriptions

Modeling with a Linear Function

If the average rate of change of a function is a constant m, a linear function f can be used to model the relation between the two variables as follows:

$$f(x) = mx + b$$

where b is the value of f at 0, that is, $f(0)$.

EXAMPLE Modeling Straight-line Depreciation

Suppose that a company just purchased some new office equipment at a cost of $2500 per machine. The company chooses to depreciate each machine using the straight-line method over 5 years.

(a) Build a linear model that expresses the value V of each machine as a function of its age, x.

(b) What is the implied domain of the function found in part (a)?

(c) Graph the linear function.

(d) What is the value of each machine after 2 years?

(e) Interpret the slope.

(f) When will the value of each machine be $500?

EXAMPLE Supply and Demand

The quantity supplied of a good is the amount of a product that a company is willing to make available for sale at a given price. The quantity demanded of a good is the amount of a product that consumers are willing to purchase at a given price. Suppose that the quantity supplied, S, and quantity demanded, D, of cellular telephones each month are given by the following functions:

$$S(p) = 60p - 900$$

$$D(p) = -15p + 2850$$

where p is the price (in dollars) of the telephone.

(a) The **equilibrium price** of a product is defined as the price at which quantity supplied equals quantity demanded. That is, the equilibrium price is the price at which $S(p) = D(p)$. Find the equilibrium price of cellular telephones. What is the equilibrium quantity, the amount demanded (or supplied), at the equilibrium price?

$$S(p) = D(p)$$

$$\boxed{p = \$50}$$

EXAMPLE Supply and Demand

$$S(p) = 60p - 900$$

$$D(p) = -15p + 2850$$

(b) Determine the prices for which quantity supplied is greater than quantity demanded. That is, solve the inequality $S(p) > D(p)$.

$$\boxed{p > \$50}$$

If the company charges more than \$50 per phone, then quantity supplied will exceed quantity demanded. In this case the company will have excess phones in inventory.

EXAMPLE Supply and Demand

$$S(p) = 60p - 900$$

$$D(p) = -15p + 2850$$

(c) Graph $S = S(p)$, $D = D(p)$ and label the equilibrium price.