# Section 4.4

# Building Quadratic Models from Verbal Descriptions and Data

# OBJECTIVE 1

✓ 1 Build Quadratic Models from Verbal Descriptions

## EXAMPLE Maximizing Revenue

The marketing department at Texas Instruments has found that, when certain calculators are sold at a price of $p$ dollars per unit, the revenue $R$ (in dollars) as a function of the price $p$ is

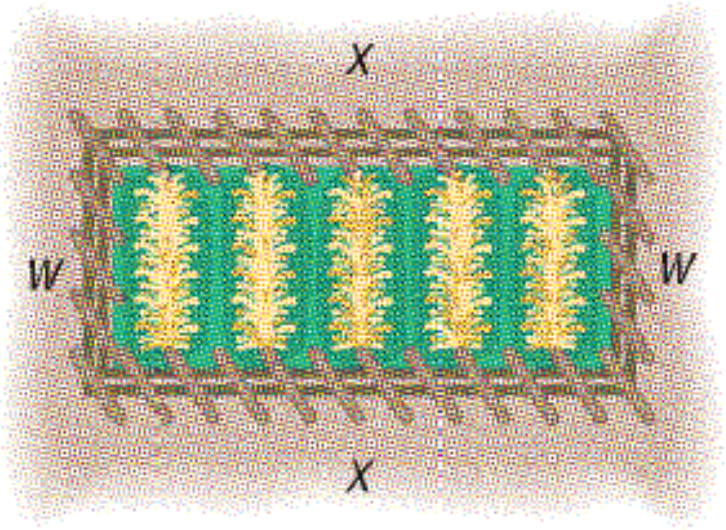
$$x = 26,000 - 160p$$

(a) Find a model that expresses the revenue $R$ as a function of the price $p$.
(b) What is the domain of $R$?
(c) What unit price should be used to maximize revenue?
(d) If this price is charged, what is the maximum revenue?
(e) How many units are sold at this price?
(f) Graph $R$.

## EXAMPLE

# Maximizing the Area Enclosed by a Fence

A farmer has 800 yards of fence to enclose a rectangular field. What are the dimensions of the rectangle that encloses the most area?



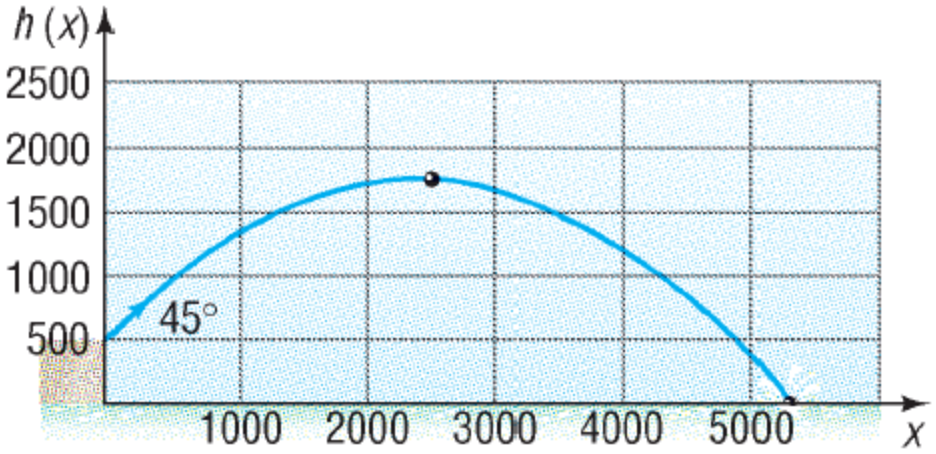
**EXAMPLE**

## Analyzing the Motion of a Projectile

A projectile is fired from a cliff 500 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 400 feet per second. In physics, it is established that the height $h$ of the projectile above the water is given by

$$h(x) = \frac{-32x^2}{(400)^2} + x + 500$$

where $x$ is the horizontal distance of the projectile from the base of the cliff.




(a) Find the maximum height of the projectile.

(b) How far from the base of the cliff will the projectile strike the water?

## Seeing the Concept

Graph

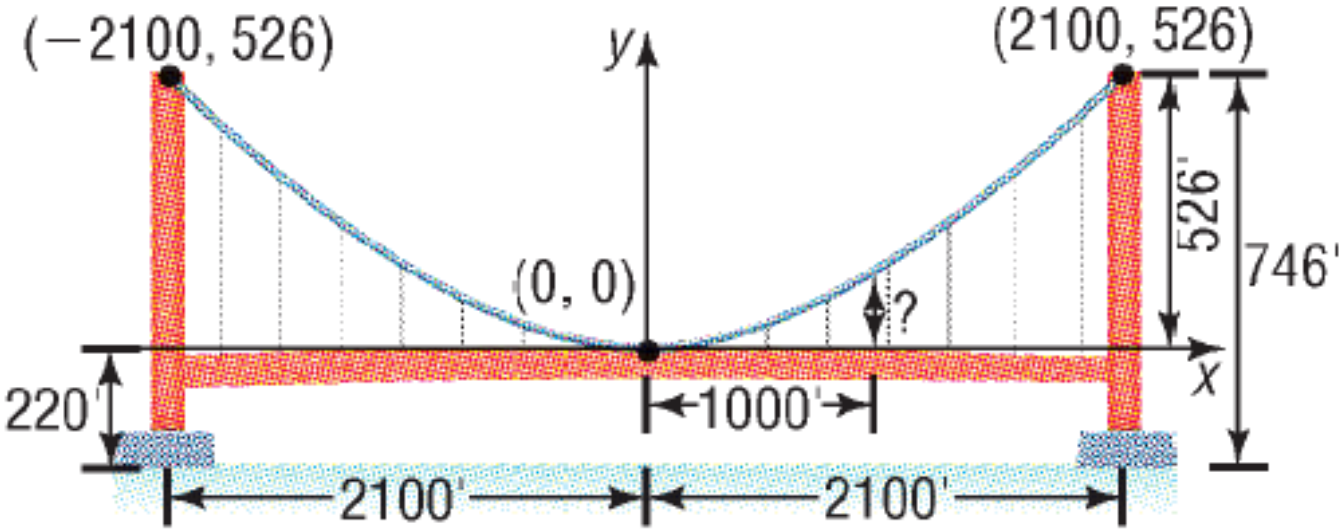
$$h(x) = \frac{-1}{5000}x^2 + x + 500$$

$0 \leq x \leq 5500$

Use MAXIMUM to find the maximum height of the projectile, and use ROOT or ZERO to find the distance from the base of the cliff to where it strikes the water. Compare your results with those obtained in Example 3.

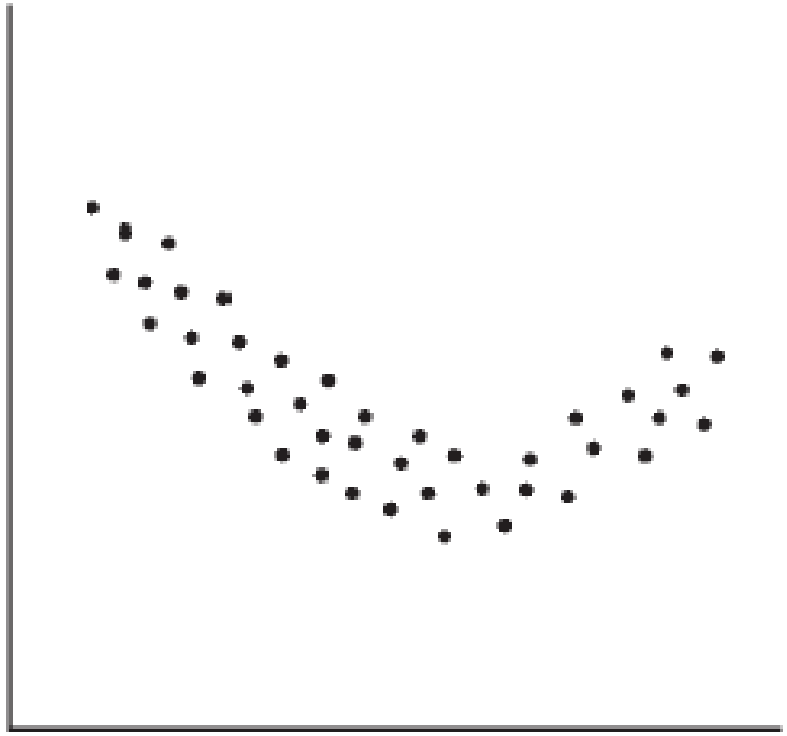
# EXAMPLE The Golden Gate Bridge

The Golden Gate Bridge, a suspension bridge, spans the entrance to San Francisco Bay. Its 746-foot-tall towers are 4200 feet apart. The bridge is suspended from two huge cables more than 3 feet in diameter; the 90-foot-wide roadway is 220 feet above the water. The cables are parabolic in shape* and touch the road surface at the center of the bridge. Find the height of the cable at a distance of 1000 feet from the center.

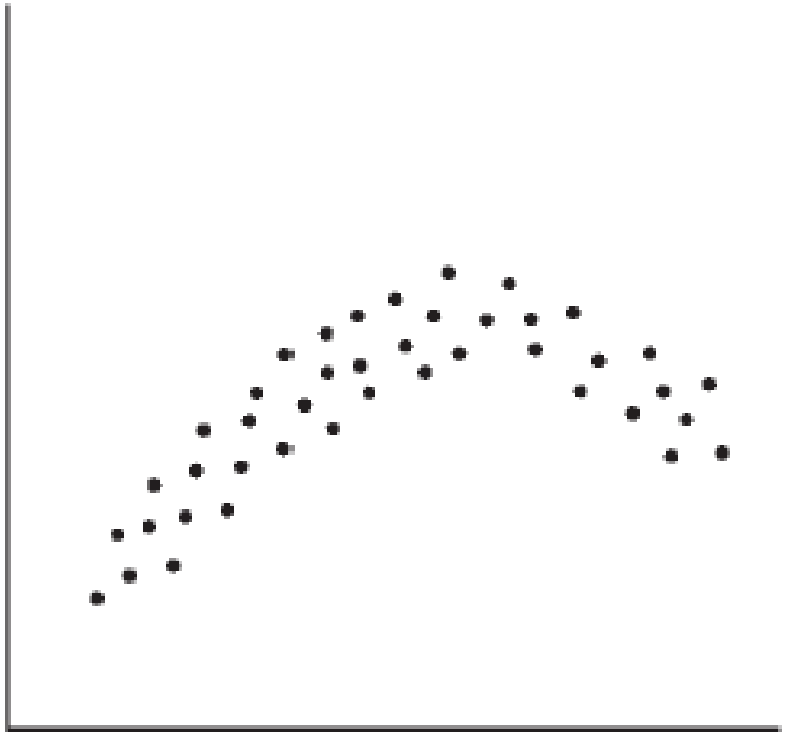


# OBJECTIVE 2

✓ 2 Build Quadratic Models from Data



$y = ax^2 + bx + c, a > 0$



$y = ax^2 + bx + c, a < 0$

**EXAMPLE**

## Fitting a Quadratic Function to Data

A farmer collected the data given in Table 9, which shows crop yields $Y$ for various amounts $x$ of fertilizer used.

(a) Use a graphing utility to draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.

(b) Use a graphing utility to find the quadratic function of best fit that models the relation between amount of fertilizer and crop yield.

(c) Use the function found in part (b) to determine the optimal amount of fertilizer to apply.

(d) Use the function found in part (b) to predict crop yield when the optimal amount of fertilizer is applied.

(e) Draw the quadratic function of best fit on the scatter diagram.

| Plot | Fertilizer, $x$ (Pounds/100 ft$^2$) | Yield (Bushels) |
|---|---|---|
| 1 | 0 | 4 |
| 2 | 0 | 6 |
| 3 | 5 | 10 |
| 4 | 5 | 7 |
| 5 | 10 | 12 |
| 6 | 10 | 10 |
| 7 | 15 | 15 |
| 8 | 15 | 17 |
| 9 | 20 | 18 |
| 10 | 20 | 21 |
| 11 | 25 | 20 |
| 12 | 25 | 21 |
| 13 | 30 | 21 |
| 14 | 30 | 22 |
| 15 | 35 | 21 |
| 16 | 35 | 20 |
| 17 | 40 | 19 |
| 18 | 40 | 19 |