Section 5.3

The Graph of a Rational Function

OBJECTIVE 1

1 Analyze the Graph of a Rational Function

EXAMPLE How to Analyze the Graph of a Rational Function

Analyze the graph of the rational function: $R(x) = \dfrac{x^2 - 4}{x^2 + 3x - 4}$

Step-by-Step Solution

STEP 1 Factor the numerator and denominator of R. Find the domain of the rational function.

STEP 2 Write R in lowest terms.

STEP 3 Locate the intercepts of the graph.

STEP 4 Test for symmetry. If $R(-x) = R(x)$, the function is even and its graph will be symmetric with respect to the y-axis. If $R(-x) = -R(x)$, the function is odd and its graph will be symmetric with respect to the origin.

STEP 5 Locate the vertical asymptotes.

STEP 6 Locate the horizontal or oblique asymptotes. Determine points, if any, at which the graph of R intersects these asymptotes.

STEP 7 Graph R using a graphing utility.

STEP 8 Use the results obtained in Steps 1 through 7 to graph R by hand.

Analyzing the Graph of a Rational Function

STEP 1: Factor the numerator and denominator of R. Find the domain of the rational function.

STEP 2: Write R in lowest terms.

STEP 3: Locate the intercepts of the graph. The x-intercepts, if any, of $R(x) = \frac{p(x)}{q(x)}$ in lowest terms satisfy the equation $p(x) = 0$. The y-intercept, if there is one, is $R(0)$.

STEP 4: Test for symmetry. Replace x by $-x$ in $R(x)$. If $R(-x) = R(x)$, there is symmetry with respect to the y-axis; if $R(-x) = -R(x)$, there is symmetry with respect to the origin.

STEP 5: Locate the vertical asymptotes. The vertical asymptotes, if any, of $R(x) = \frac{p(x)}{q(x)}$ in lowest terms are found by identifying the real zeros of $q(x)$. Each zero of the denominator gives rise to a vertical asymptote.

STEP 6: Locate the horizontal or oblique asymptotes, if any, using the procedure given in Section 4.2. Determine points, if any, at which the graph of R intersects these asymptotes.

STEP 7: Graph R using a graphing utility.

STEP 8: Use the results obtained in Steps 1 through 7 to graph R by hand.

EXAMPLE

Analyzing the Graph of a Rational Function

Analyze the graph of the rational function:

$$R(x) = \frac{x^2 + 3x + 2}{x}$$

EXAMPLE

Analyzing the Graph of a Rational Function

Analyze the graph of the rational function:

$$R(x) = \frac{x^2 - x + 12}{x^2 - 1}$$

EXAMPLE

Analyzing the Graph of a Rational Function with a Hole

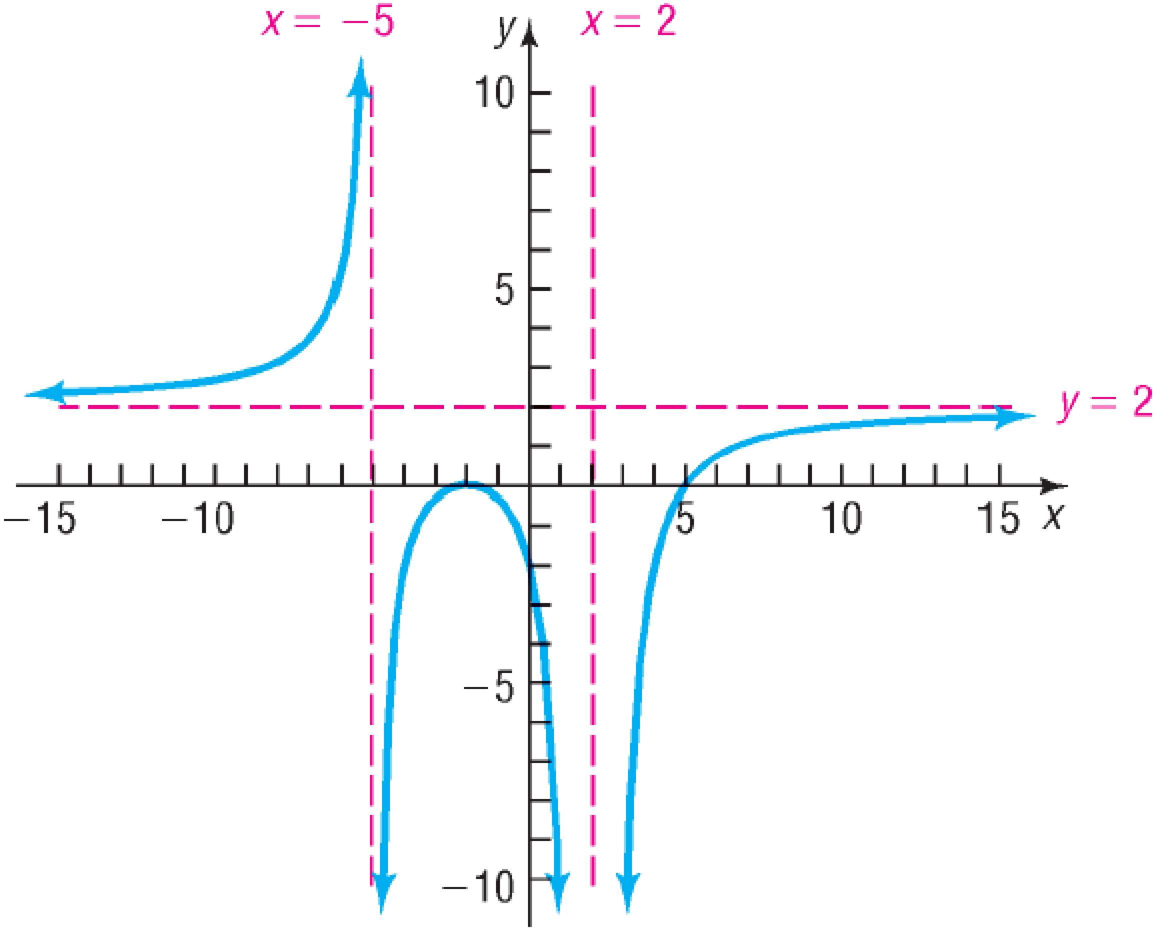
Analyze the graph of the rational function:

$$R(x) = \frac{x^2 - 9}{x^2 + 9x + 18}$$

EXAMPLE

Constructing a Rational Function from Its Graph

Make up a rational function that might have the graph shown in Figure



OBJECTIVE 2

2 Solve Applied Problems Involving Rational Functions

EXAMPLE Finding the Least Cost of a Can

Reynolds Metal Company manufactures aluminum cans in the shape of a cylinder with a capacity of 500 cubic centimeters $\left(\frac{1}{2}\text{ liter}\right)$. The top and bottom of the can are made of a special aluminum alloy that costs 0.05¢ per square centimeter. The sides of the can are made of material that costs 0.02¢ per square centimeter.

(a) Express the cost of material for the can as a function of the radius r of the can.

(b) Use a graphing utility to graph the function $C = C(r)$.

(c) What value of r will result in the least cost?

(d) What is this least cost?