Section 5.4

Polynomial and Rational Inequalities

OBJECTIVE 1

1 Solve Polynomial Inequalities Algebraically and Graphically

EXAMPLE

Solving a Polynomial Inequality Using Its Graph

Solve $(x+3)^2(x-1)(x-4) \leq 0$

EXAMPLE **How to Solve a Polynomial Inequality Algebraically**

Solve the inequality $(x+3)^2(x-1)(x-4) \le 0$ algebraically, and graph the solution set.

Step-by-Step Solution

STEP 1 Write the inequality so that a polynomial expression f is on the left side and zero is on the right side.

STEP 2 Determine the real zeros (x-intercepts of the graph) of f.

STEP 3 Use the zeros found in Step 2 to divide the real number line into intervals.

STEP 4 Select a number in each interval, evaluate f at the number, and determine whether f is positive or negative. If f is positive, all values of x in the interval are positive. If f is negative, all values of x in the interval are negative.

Note: If the inequality is not strict ($\le$ or $\ge$), include the solutions of $f(x) = 0$ in the solution set.

OBJECTIVE 2

2 Solve Rational Inequalities Algebraically and Graphically

EXAMPLE Solving a Rational Inequality Using Its Graph

Solve $\dfrac{x-4}{x+2} \geq 2$ by graphing.

EXAMPLE **How to Solve a Rational Inequality Algebraically**

Solve the inequality $\dfrac{x-4}{x+2} \geq 2$ algebraically, and graph the solution set.

Step-by-Step Solution

STEP 1 Write the inequality so that a rational expression f is on the left side and zero is on the right side.

STEP 2 Determine the real zeros (x-intercepts of the graph) of f and the real numbers for which f is undefined.

STEP 3 Use the zeros and undefined values found in Step 2 to divide the real number line into intervals.

STEP 4 Select a number in each interval, evaluate f at the number, and determine whether f is positive or negative. If f is positive, all values of f in the interval are positive. If f is negative, all values of f in the interval are negative.

Note: If the inequality is not strict ($\leq$ or $\geq$), include the solutions of $f(x) = 0$ in the solution set.

Steps for Solving Polynomial and Rational Inequalities Algebraically

Step 1: Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \qquad f(x) \geq 0 \qquad f(x) < 0 \qquad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient.

Step 2: Determine the numbers at which the expression f on the left side equals zero and, if the expression is rational, the numbers at which the expression f on the left side is undefined.

Step 3: Use the numbers found in Step 2 to separate the real number line into intervals.

Step 4: Select a number in each interval and evaluate f at the number.

(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.

(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.

If the inequality is not strict ($\geq$ or $\leq$), include the solutions of $f(x) = 0$ in the solution set, but be careful not to include values of x where the expression is undefined.