Section 5.4
Polynomial and Rational Inequalities
OBJECTIVE 1

✓ Solve Polynomial Inequalities Algebraically and Graphically
EXAMPLE

Solving a Polynomial Inequality Using Its Graph

Solve \((x + 3)^2 (x - 1)(x - 4) \leq 0\)
Solve the inequality \((x + 3)^2 (x - 1)(x - 4) \leq 0\) algebraically, and graph the solution set.

**Step-by-Step Solution**

**Step 1** Write the inequality so that a polynomial expression \(f\) is on the left side and zero is on the right side.

**Step 2** Determine the real zeros (x-intercepts of the graph) of \(f\).

**Step 3** Use the zeros found in Step 2 to divide the real number line into intervals.

**Step 4** Select a number in each interval, evaluate \(f\) at the number, and determine whether \(f\) is positive or negative. If \(f\) is positive, all values of \(x\) in the interval are positive. If \(f\) is negative, all values of \(x\) in the interval are negative.

**Note:** If the inequality is not strict (\(\leq\) or \(\geq\)), include the solutions of \(f(x) = 0\) in the solution set.
OBJECTIVE 2

Solve Rational Inequalities Algebraically and Graphically
Solve \( \frac{x-4}{x+2} \geq 2 \) by graphing.
How to Solve a Rational Inequality Algebraically

Solve the inequality \( \frac{x - 4}{x + 2} \geq 2 \) algebraically, and graph the solution set.

**Step-by-Step Solution**

**Step 1** Write the inequality so that a rational expression \( f \) is on the left side and zero is on the right side.

**Step 2** Determine the real zeros (x-intercepts of the graph) of \( f \) and the real numbers for which \( f \) is undefined.

**Step 3** Use the zeros and undefined values found in Step 2 to divide the real number line into intervals.

**Step 4** Select a number in each interval, evaluate \( f \) at the number, and determine whether \( f \) is positive or negative. If \( f \) is positive, all values of \( f \) in the interval are positive. If \( f \) is negative, all values of \( f \) in the interval are negative.

**Note:** If the inequality is not strict (\( \leq \) or \( \geq \)), include the solutions of \( f(x) = 0 \) in the solution set.
Steps for Solving Polynomial and Rational Inequalities Algebraically

**Step 1:** Write the inequality so that a polynomial or rational expression \( f \) is on the left side and zero is on the right side in one of the following forms:

\[
 f(x) > 0 \quad f(x) \geq 0 \quad f(x) < 0 \quad f(x) \leq 0
\]

For rational expressions, be sure that the left side is written as a single quotient.

**Step 2:** Determine the numbers at which the expression \( f \) on the left side equals zero and, if the expression is rational, the numbers at which the expression \( f \) on the left side is undefined.

**Step 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

**Step 4:** Select a number in each interval and evaluate \( f \) at the number.

(a) If the value of \( f \) is positive, then \( f(x) > 0 \) for all numbers \( x \) in the interval.

(b) If the value of \( f \) is negative, then \( f(x) < 0 \) for all numbers \( x \) in the interval.

If the inequality is not strict (\( \geq \) or \( \leq \)), include the solutions of \( f(x) = 0 \) in the solution set, but be careful not to include values of \( x \) where the expression is undefined.