Section 6.5

Properties of Logarithms

OBJECTIVE 1

1 Work with the Properties of Logarithms

EXAMPLE

Establishing Properties of Logarithms

(a) Show that $\log_a 1 = 0$.

(b) Show that $\log_a a = 1$.

$$\boxed{\log_a 1 = 0 \qquad \log_a a = 1}$$

THEOREM

Properties of Logarithms

In the properties given next, M and a are positive real numbers, $a \neq 1$, and r is any real number.

The number $\log_a M$ is the exponent to which a must be raised to obtain M. That is,

$$a^{\log_a M} = M \tag{1}$$

The logarithm to the base a of a raised to a power equals that power. That is,

$$\log_a a^r = r \tag{2}$$

EXAMPLE

Using Properties (1) and (2)

(a) $3^{\log_3 18}$ (b) $2^{\log_2(-5)}$ (c) $\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{20}$ (d) $\ln e^3$

THEOREM

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, with $a \neq 1$, and r is any real number.

The Log of a Product Equals the Sum of the Logs

$$\log_a(MN) = \log_a M + \log_a N \quad \textbf{(3)}$$

The Log of a Quotient Equals the Difference of the Logs

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \quad \textbf{(4)}$$

The Log of a Power Equals the Product of the Power and the Log

$$\log_a M^r = r \log_a M \quad \textbf{(5)}$$

OBJECTIVE 2

2 Write a Logarithmic Expression as a Sum or Difference of Logarithms

EXAMPLE

Writing a Logarithmic Expression as a Sum of Logarithms

Write $\log_2\left(x^2\sqrt[3]{x-1}\right)$, $x>1$, as a sum of logarithms.

Express all powers as factors.

EXAMPLE

Writing a Logarithmic Expression as a Difference of Logarithms

Write $\log_6 \dfrac{x^4}{\left(x^2+3\right)^2}$, $x \neq 0$, as a difference of logarithms.

Express all powers as factors.

EXAMPLE

Writing a Logarithmic Expression as a Sum and Difference of Logarithms

Write $\ln \dfrac{x^3\sqrt{x-2}}{(x+1)^2}$, $x > 2$, as a sum and difference of logarithms.

Express all powers as factors.

OBJECTIVE 3

3 Write a Logarithmic Expression as a Single Logarithm

EXAMPLE

Writing Expressions as a Single Logarithm

Write each of the following as a single logarithm.

$$(a)\ 3\ln 2+\ln\left(x^2\right)+2$$

$$(b)\ \frac{1}{2}\log_a 4-2\log_a 5$$

$$(c)\ -2\log_a 3+3\log_a 2-\log_a\left(x^2+1\right)$$

Theorem

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, with $a \neq 1$.

$$\text{If } M = N, \text{ then } \log_a M = \log_a N.$$

$$\text{If } \log_a M = \log_a N, \text{ then } M = N.$$

OBJECTIVE 4

4 Evaluate Logarithms Whose Base Is Neither 10 nor *e*

EXAMPLE

Approximating a Logarithm Whose Base Is Neither 10 nor e

Approximate $\log_3 12$. Round answer to four decimal places.

Theorem

Change-of-Base Formula

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then

$$\boxed{\log_a M = \frac{\log_b M}{\log_b a}}$$

$$\boxed{\log_a M = \frac{\log M}{\log a} \quad \text{and} \quad \log_a M = \frac{\ln M}{\ln a}}$$

EXAMPLE

Using the Change-of-Base Formula

Approximate: (a) $\log_7 35$ (b) $\log_{\frac{1}{3}} \sqrt{2}$

OBJECTIVE 5

5 Graph Logarithmic Functions Whose Base Is Neither 10 Nor *e*

EXAMPLE

Graphing a Logarithmic Function Whose Base Is Neither 10 Nor e

Use a graphing utility to graph $y = \log_5 x$.

Summary

Properties of Logarithms

In the list that follows, $a > 0$, $a \neq 1$, and $b > 0$, $b \neq 1$; also, $M > 0$ and $N > 0$.

Definition	$y = \log_a x$ means $x = a^y$
Properties of logarithms	$\log_a 1 = 0; \quad \log_a a = 1$
	$a^{\log_a M} = M; \quad \log_a a^r = r$
	$\log_a(MN) = \log_a M + \log_a N$
	$\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$
	$\log_a M^r = r \log_a M$
	If $M = N$, then $\log_a M = \log_a N$.
	If $\log_a M = \log_a N$, then $M = N$.
Change-of-Base Formula	$\log_a M = \dfrac{\log_b M}{\log_b a}$