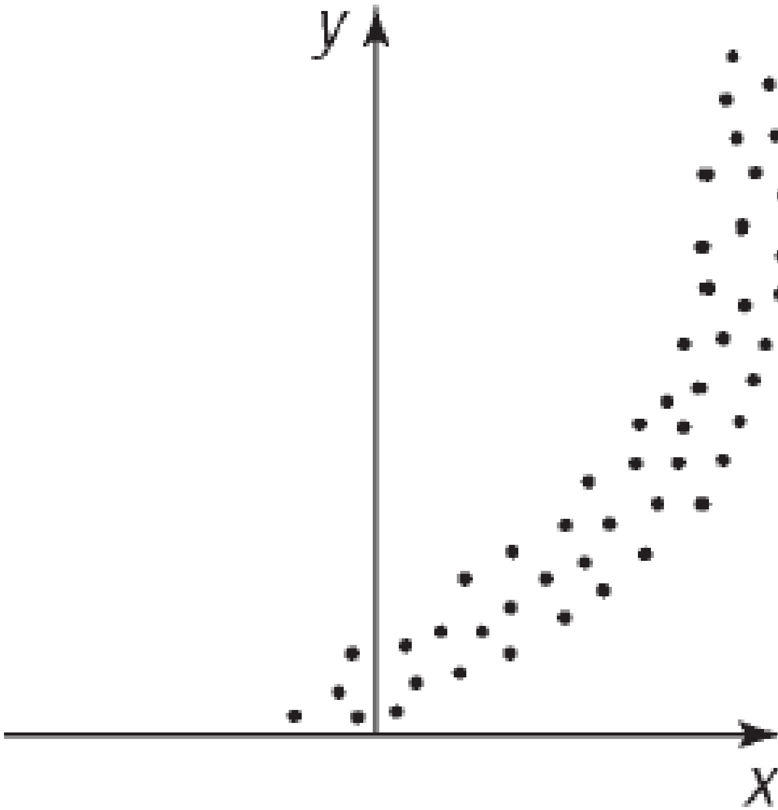
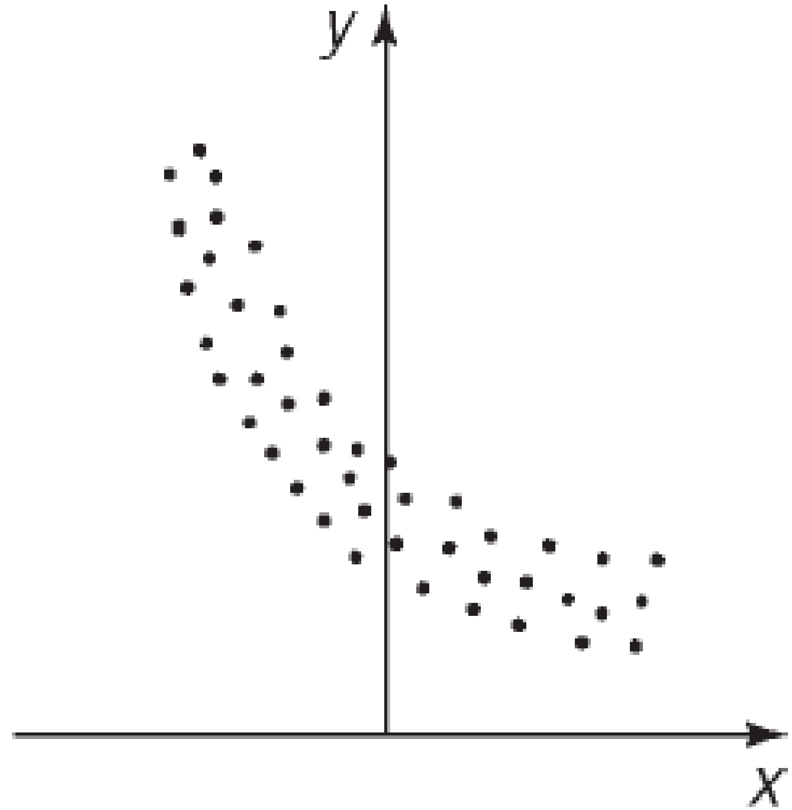
Section 6.9

Building Exponential, Logarithmic, and Logistic Models from Data



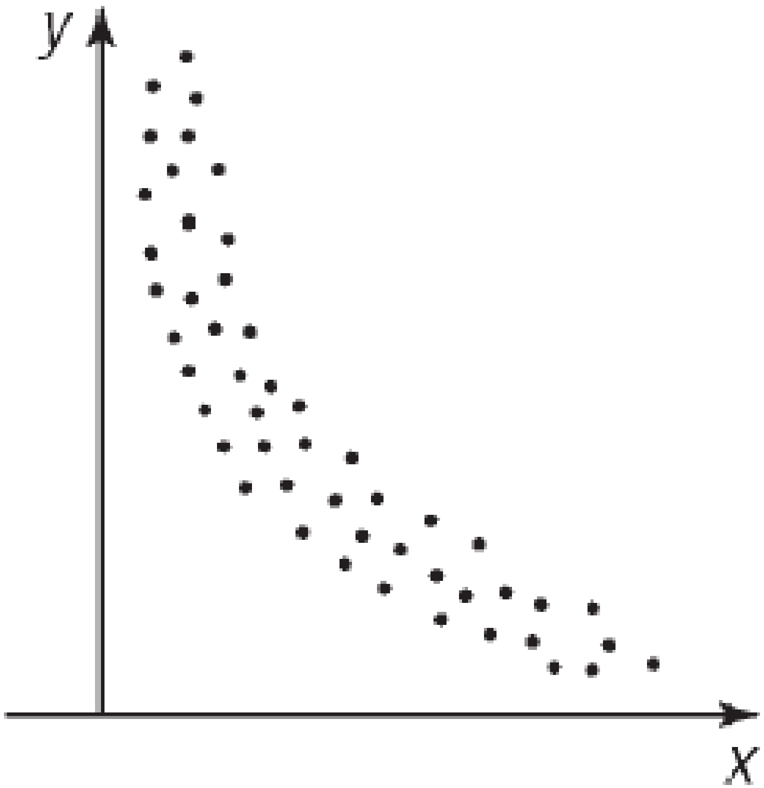
$y = ab^x, a > 0, b > 1$

Exponential



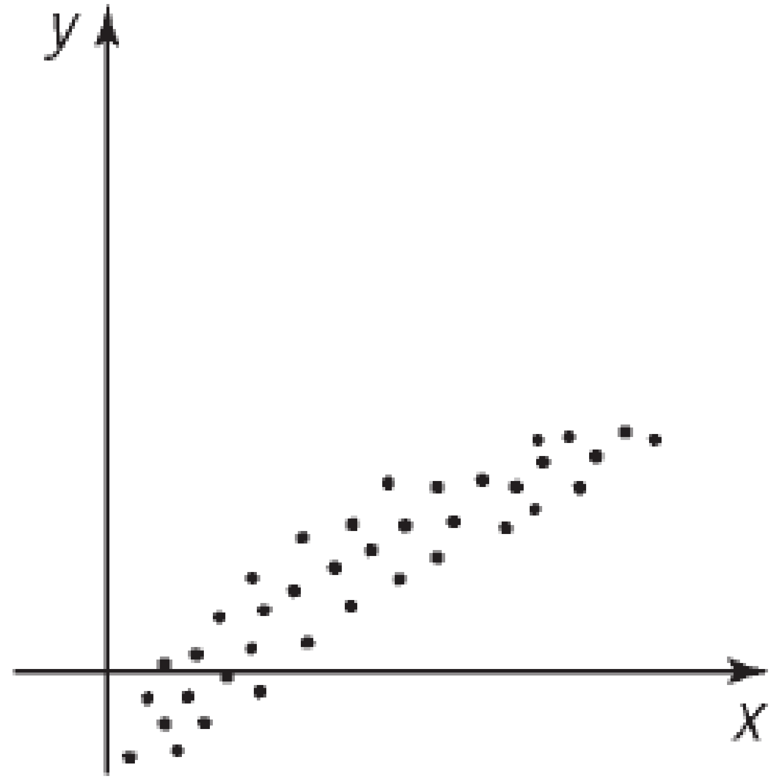
$y = ab^x, 0 < b < 1, a > 0$

Exponential



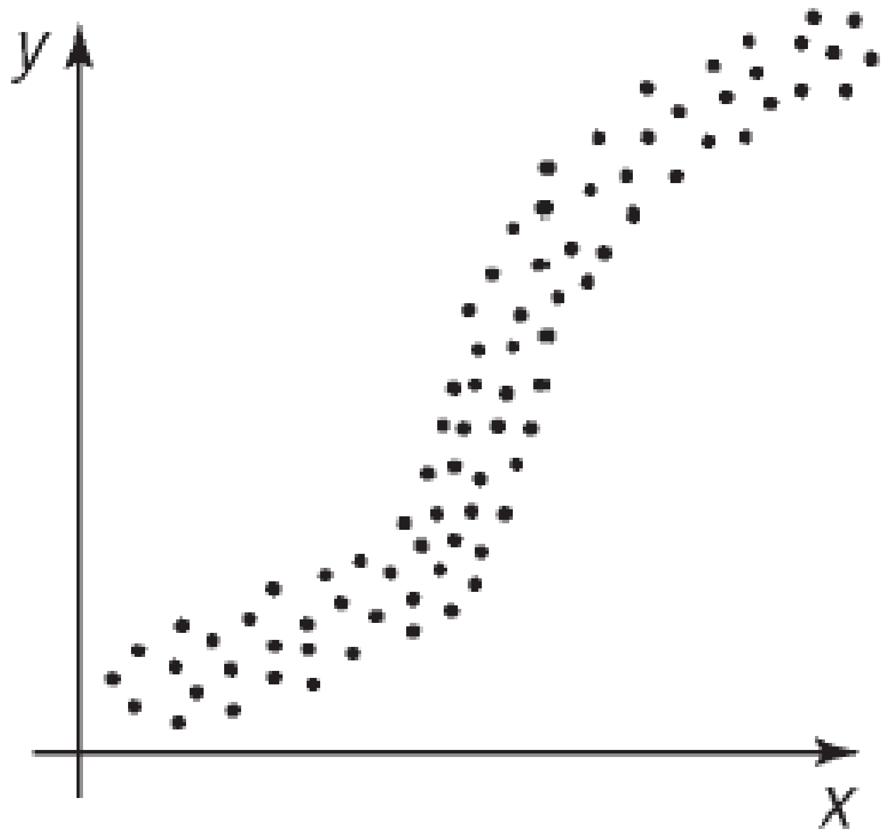
$y = a + b \ln x,\ a > 0,\ b < 0$

Logarithmic



$y = a + b \ln x,\ a > 0,\ b > 0$

Logarithmic



$$y = \frac{c}{1 + ae^{-bx}}, \ a > 0, \ b > 0, c > 0$$

Logistic

OBJECTIVE 1

1 Build an Exponential Model from Data

EXAMPLE Fitting an Exponential Function to Data

Kathleen is interested in finding a function that explains the growth of cell phone usage in the United States. She gathers data on the number (in millions) of U.S. cell phone subscribers from 1985 through 2005. The data are shown in Table 9.

(a) Using a graphing utility, draw a scatter diagram with year as the independent variable.

(b) Using a graphing utility, fit an exponential function to the data.

(c) Express the function found in part (b) in the form $A = A_0e^{kt}$.

(d) Graph the exponential function found in part (b) or (c) on the scatter diagram.

(e) Using the solution to part (b) or (c), predict the number of U.S. cell phone subscribers in 2009.

(f) Interpret the value of k found in part (c).

Table on next slide

Year, x	Number of Subscribers (in millions), y
1985 ($x = 1$)	0.34
1986 ($x = 2$)	0.68
1987 ($x = 3$)	1.23
1988 ($x = 4$)	2.07
1989 ($x = 5$)	3.51
1990 ($x = 6$)	5.28
1991 ($x = 7$)	7.56
1992 ($x = 8$)	11.03
1993 ($x = 9$)	16.01
1994 ($x = 10$)	24.13
1995 ($x = 11$)	33.76
1996 ($x = 12$)	44.04
1997 ($x = 13$)	55.31
1998 ($x = 14$)	69.21
1999 ($x = 15$)	86.05
2000 ($x = 16$)	109.48
2001 ($x = 17$)	128.37
2002 ($x = 18$)	140.77
2003 ($x = 19$)	158.72
2004 ($x = 20$)	182.14
2005 ($x = 21$)	207.90

OBJECTIVE 2

2 Build a Logarithmic Model from Data

EXAMPLE Fitting a Logarithmic Function to Data

Jodi, a meteorologist, is interested in finding a function that explains the relation between the height of a weather balloon (in kilometers) and the atmospheric pressure (measured in millimeters of mercury) on the balloon. She collects the data shown in Table 11.

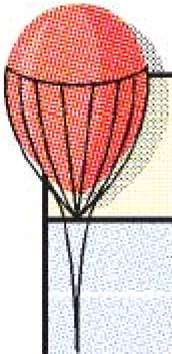
(a) Using a graphing utility, draw a scatter diagram of the data with atmospheric pressure as the independent variable.

(b) It is known that the relation between atmospheric pressure and height follows a logarithmic model. Using a graphing utility, fit a logarithmic function to the data.

(c) Draw the logarithmic function found in part (b) on the scatter diagram.

(d) Use the function found in part (b) to predict the height of the weather balloon if the atmospheric pressure is 560 millimeters of mercury.

Table on next slide



Atmospheric Pressure, *p*	Height, *h*
760	0
740	0.184
725	0.328
700	0.565
650	1.079
630	1.291
600	1.634
580	1.862
550	2.235

OBJECTIVE 3

3 Build a Logistic Model from Data

EXAMPLE Fitting a Logistic Function to Data

The data in Table 11 represent the amount of yeast biomass in a culture after t hours.

(a) Using a graphing utility, draw a scatter diagram of the data with time as the independent variable.

(b) Using a graphing utility, fit a logistic function to the data.

(c) Using a graphing utility, graph the function found in part (b) on the scatter diagram.

(d) What is the predicted carrying capacity of the culture?

(e) Use the function found in part (b) to predict the population of the culture at $t = 19$ hours.

Table on next slide

Time (in hours)	Yeast Biomass	Time (in hours)	Yeast Biomass
0	9.6	10	513.3
1	18.3	11	559.7
2	29.0	12	594.8
3	47.2	13	629.4
4	71.1	14	640.8
5	119.1	15	651.1
6	174.6	16	655.9
7	257.3	17	659.6
8	350.7	18	661.8
9	441.0		