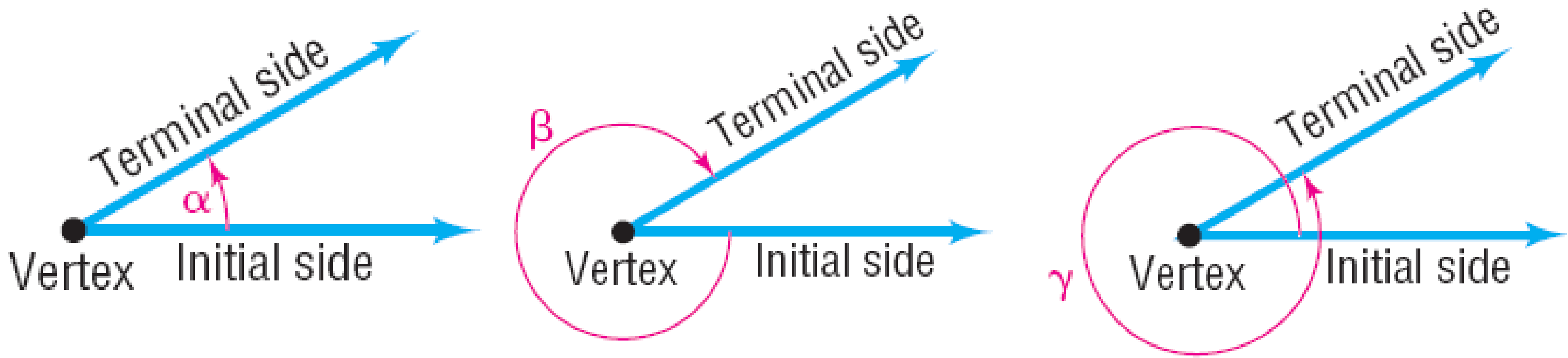
Section 7.1

Angles and Their Measure



Counterclockwise rotation

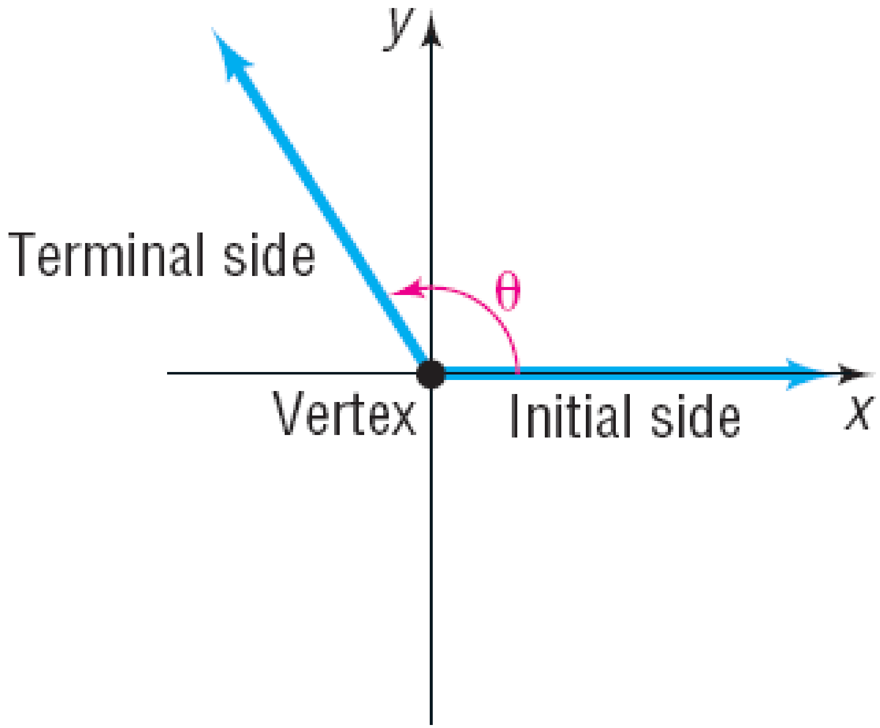
Positive angle

Clockwise rotation

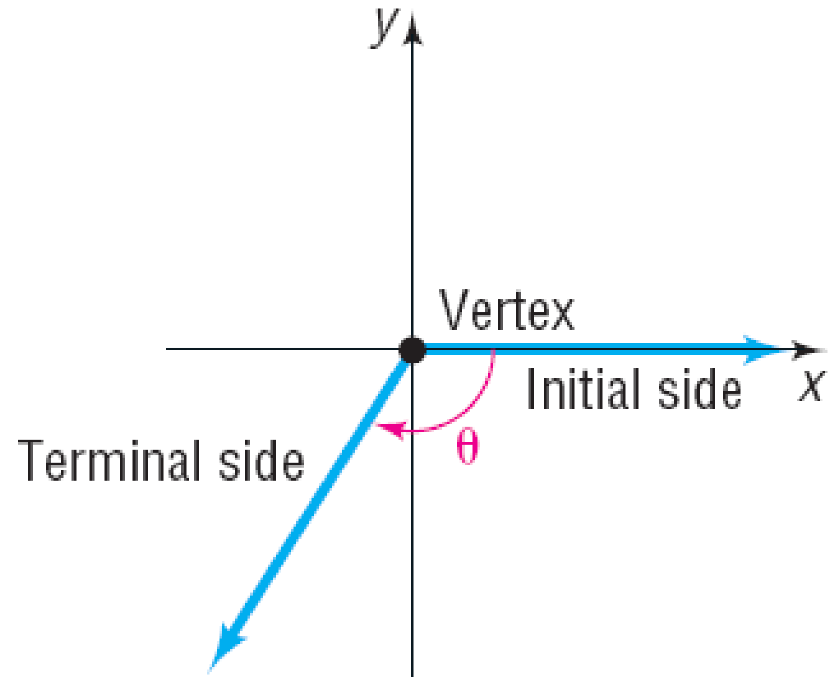
Negative angle

Counterclockwise rotation

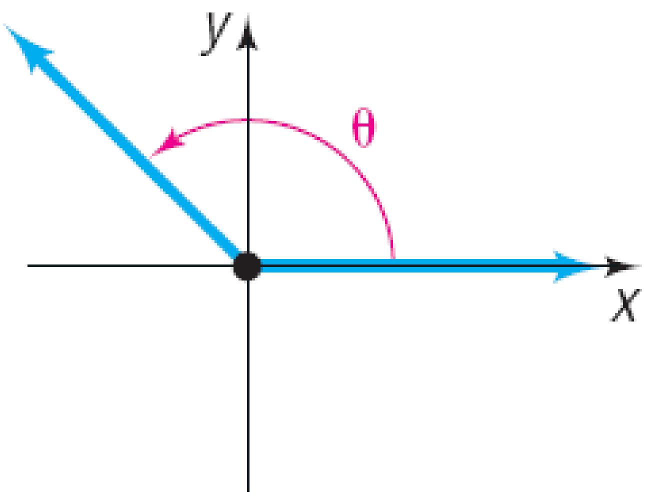
Positive angle



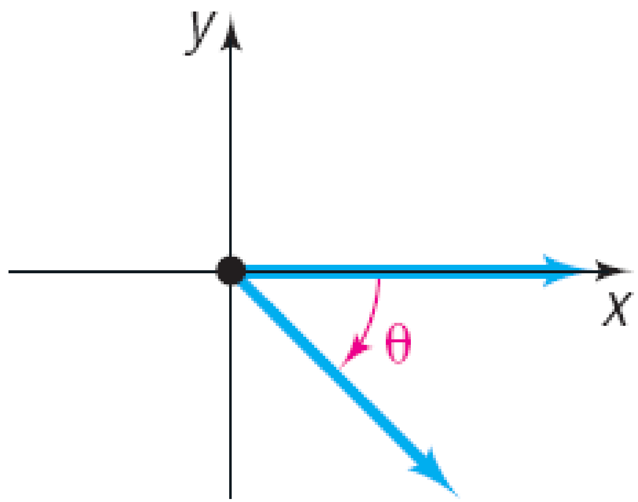
(a) θ is in standard position;
θ is positive



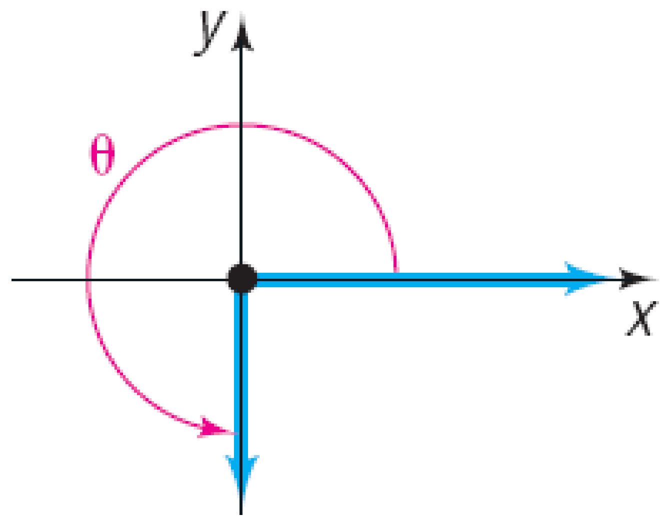
(b) θ is in standard position;
θ is negative



(a) θ lies in quadrant II



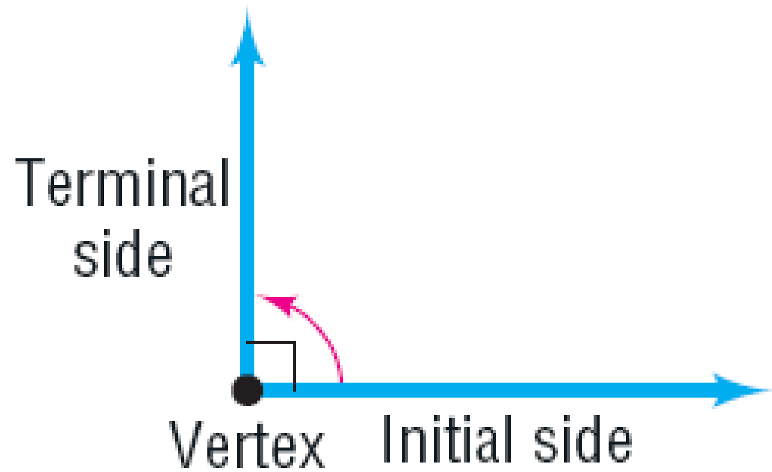
(b) θ lies in quadrant IV



(c) θ is a quadrantal angle

Degrees

(a) 1 revolution counterclockwise, 360°



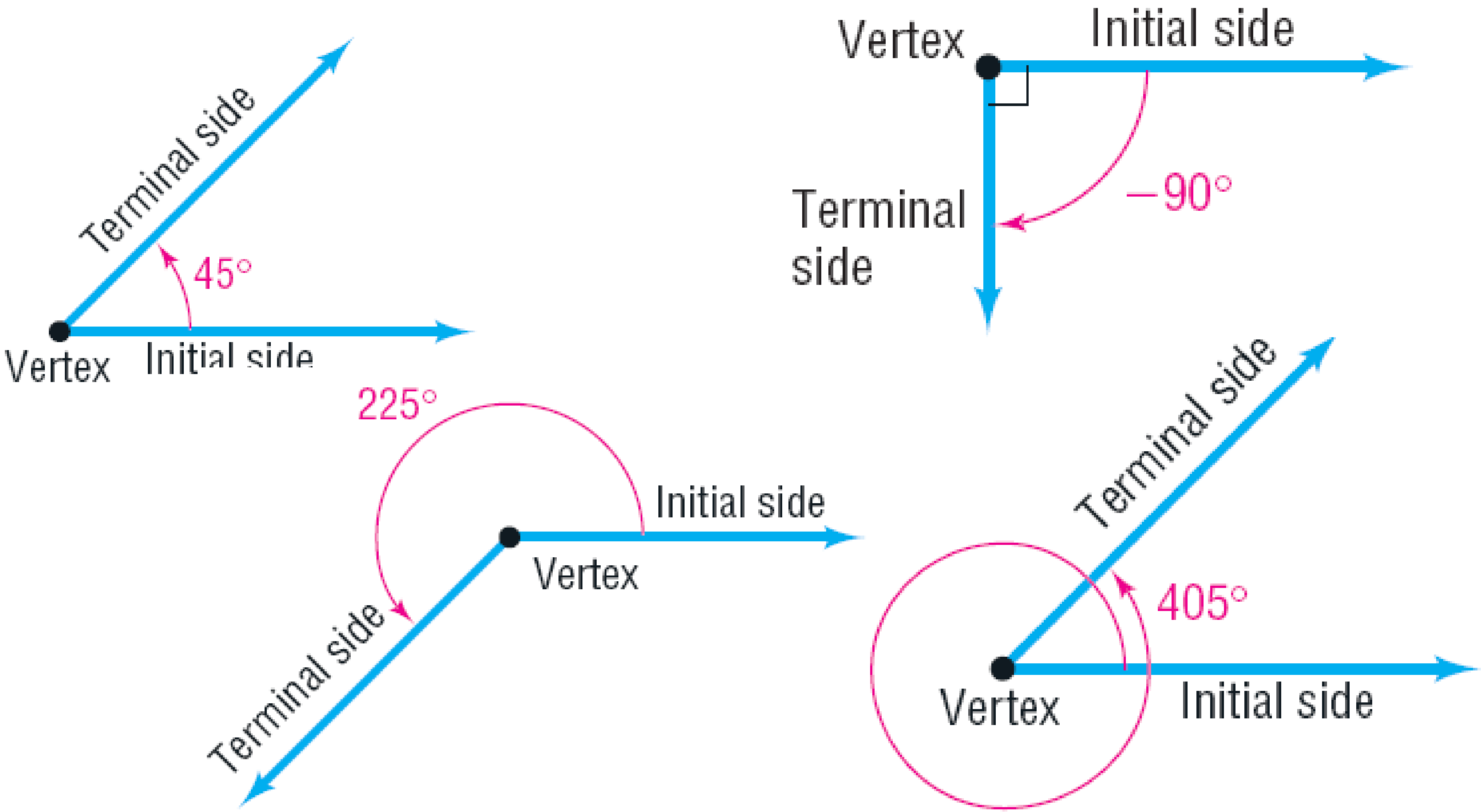
(b) right angle, $\frac{1}{4}$ revolution counter-clockwise, 90°

(c) straight angle, $\frac{1}{2}$ revolution counter-clockwise, 180°

EXAMPLE **Drawing an Angle**

Draw each angle.

(a) 45° (b) -90° (c) 225° (d) 405°



OBJECTIVE 1

1 Convert between Decimals and Degrees, Minutes, Seconds Forms for Angles

$$1 \text{ counterclockwise revolution} = 360°$$

$$1° = 60' \qquad 1' = 60''$$

EXAMPLE

Converting between Degrees, Minutes, Seconds Form and Decimal Form

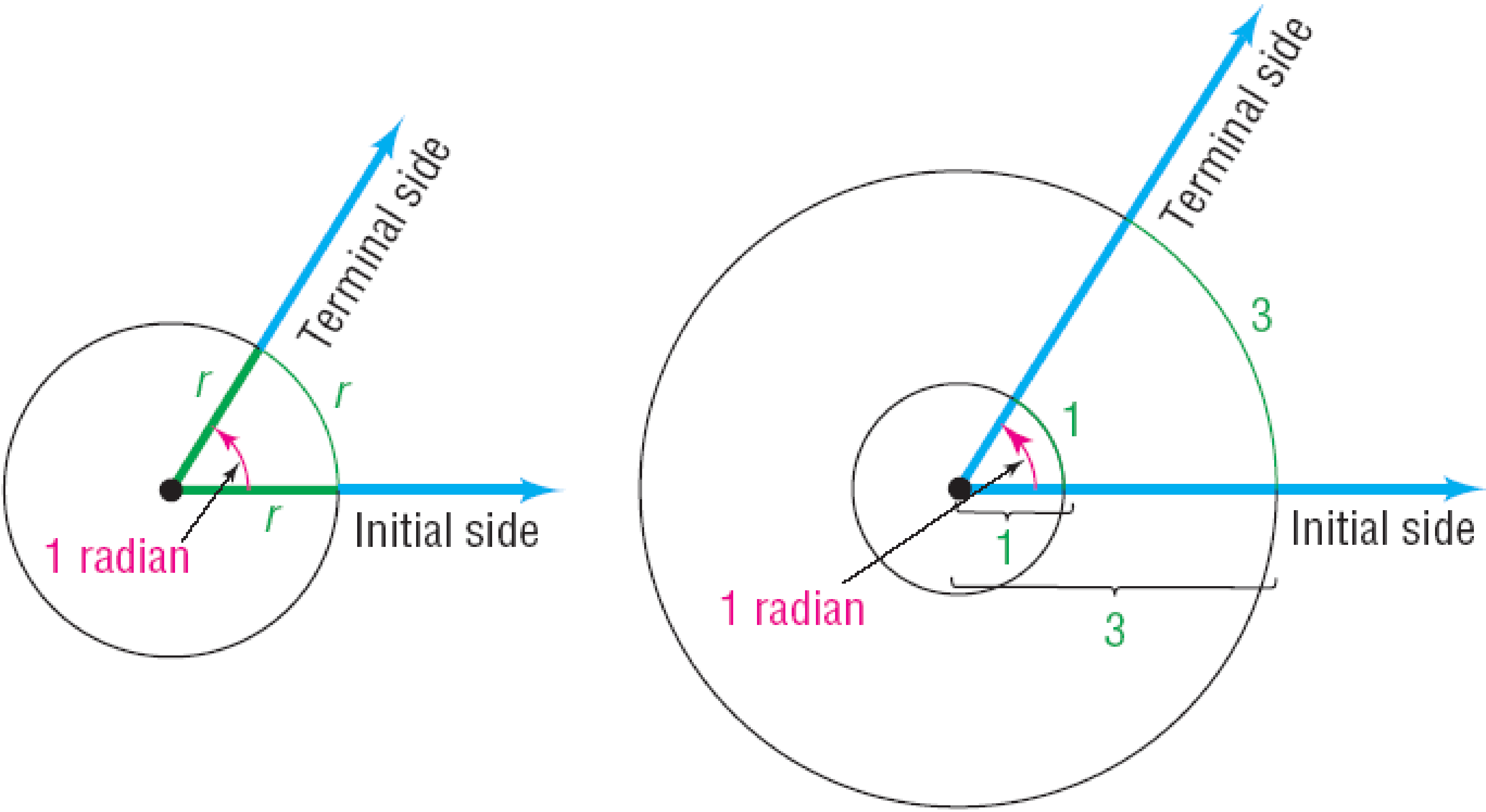
(a) Convert 45°10'15" to decimal in degrees.

Round the answer to four decimal places.

(b) Convert 21.256° to the D°M'S" form.

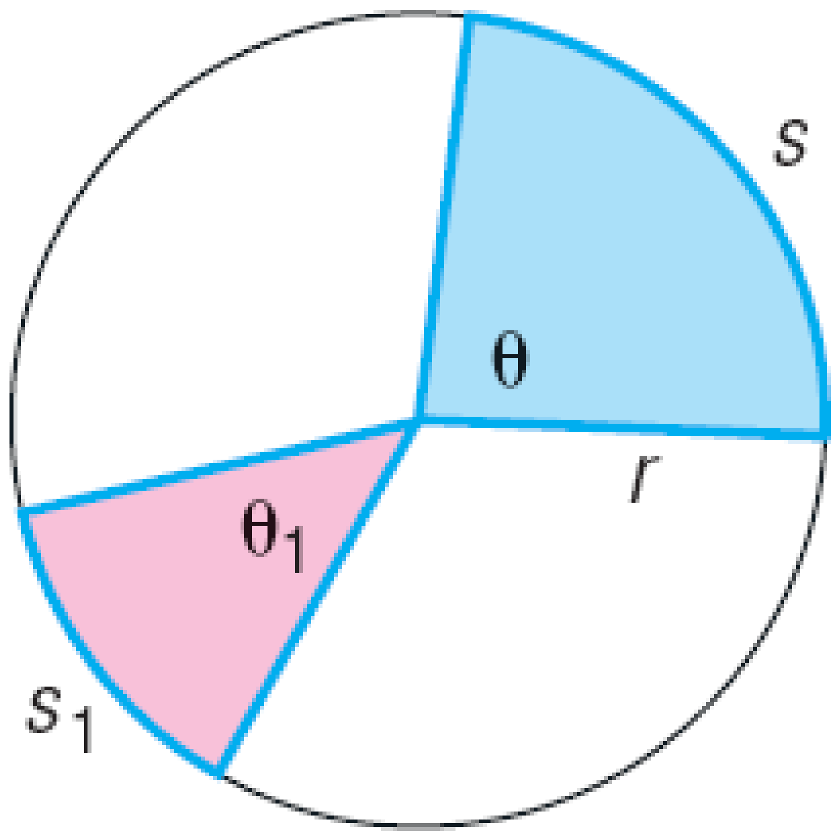
Round the answer to the nearest second.

Radians


Terminal side
r
r
r
1 radian
Initial side
Terminal side
3
1
1
3
1 radian
Initial side

OBJECTIVE 2

2 Find the Arc Length of a Circle



$$\frac{\theta}{\theta_1} = \frac{s}{s_1}$$

Theorem

Arc Length

For a circle of radius r, a central angle of θ radians subtends an arc whose length s is

$$\boxed{s = r\theta}$$

EXAMPLE

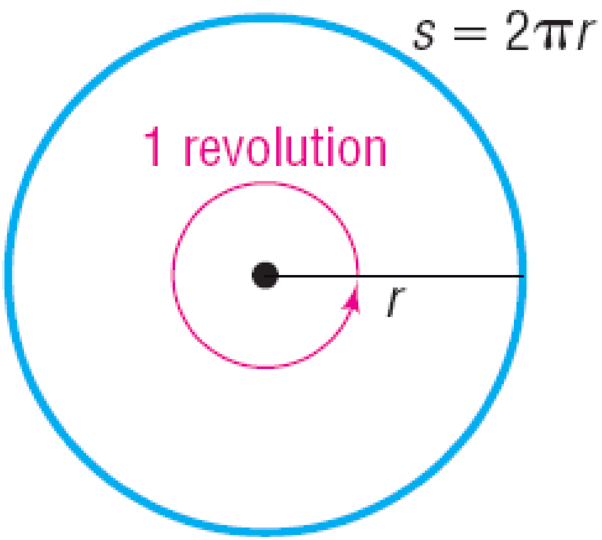
Finding the Length of an Arc of a Circle

Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.5 radian.

$$\boxed{s = r\theta}$$

OBJECTIVE 3

3 Convert from Degrees to Radians and from Radians to Degrees



$$\boxed{1 \text{ revolution} = 2\pi \text{ radians}}$$

$$\boxed{180^\circ = \pi \text{ radians}}$$

$$\boxed{1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}}$$

EXAMPLE Converting from Degrees to Radians

Convert each angle in degrees to radians.

(a) 80 (b) 140 (c) -30 (d) 100

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

EXAMPLE Converting Radians to Degrees

Convert each angle in radians to degrees.

(a) $\dfrac{2\pi}{3}$ radians

(b) $\dfrac{5\pi}{6}$ radians

(c) $\dfrac{-3\pi}{5}$ radians

(d) $\dfrac{8\pi}{3}$ radians

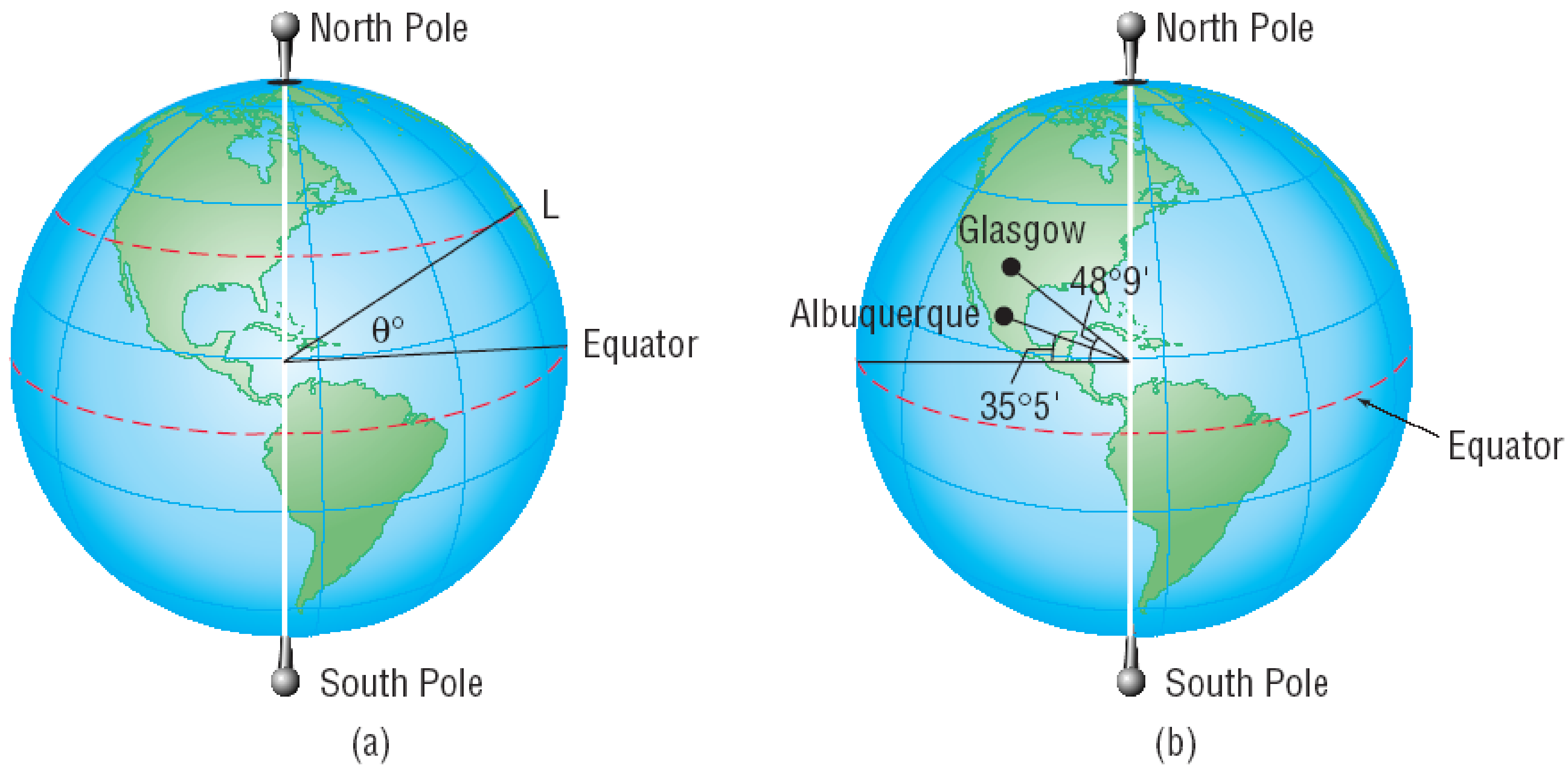
(e) 2 radians

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

EXAMPLE **Finding the Distance between Two Cities**

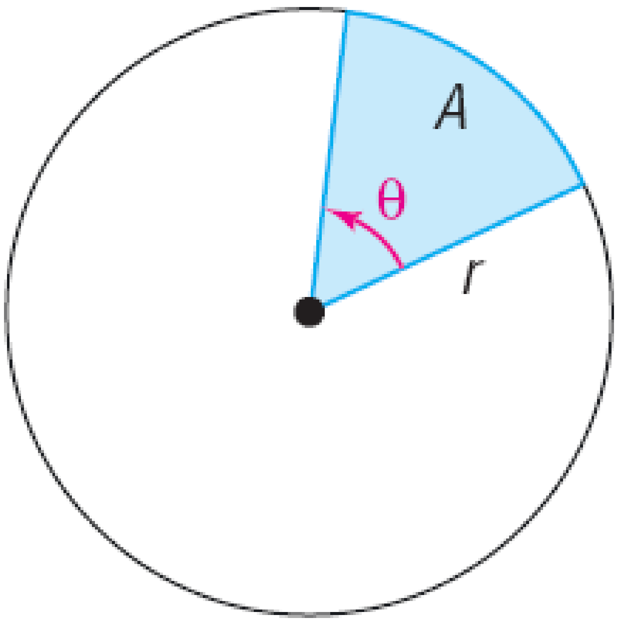
See Figure 13(a). The latitude of a location L is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to L. See Figure 13(b). Glasgow, Montana, is due north of Albuquerque, New Mexico. Find the distance between Glasgow ($48°9'$ north latitude) and Albuquerque ($35°5'$ north latitude). Assume that the radius of Earth is 3960 miles.



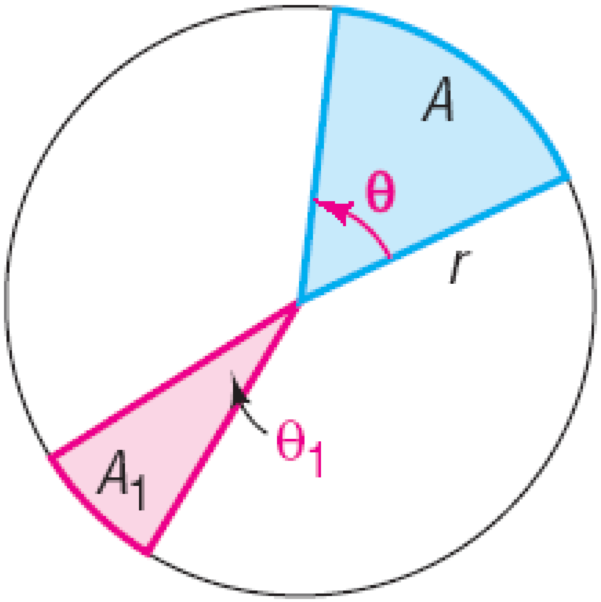
OBJECTIVE 4

4 Find the Area of a Sector of a Circle

Area of a Sector



$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$



The area A of the sector of a circle of radius r formed by a central angle of θ radians is

$$\boxed{A = \frac{1}{2} r^2 \theta}$$

EXAMPLE

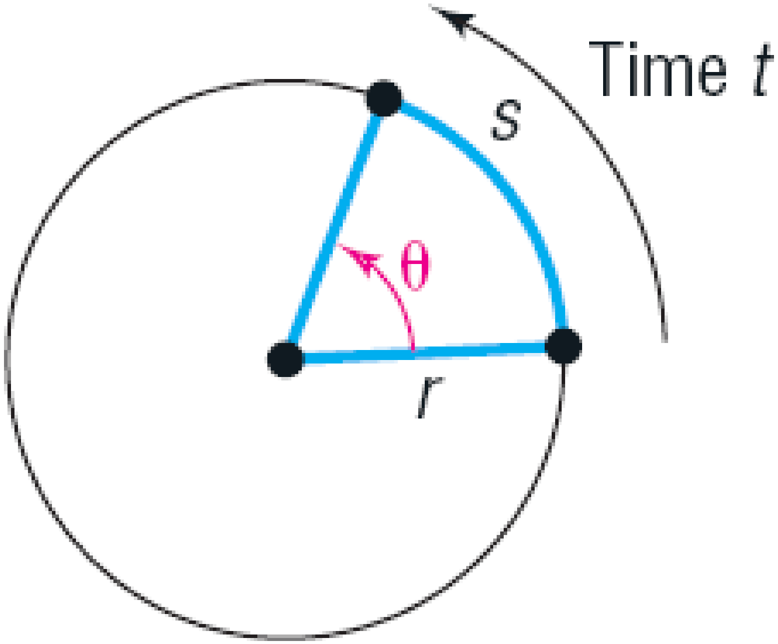
Finding the Area of a Sector of a Circle

Find the area of the sector of a circle of radius 5 feet formed by an angle of 40 . Round the answer to two decimal places.

$$\boxed{A = \frac{1}{2} r^2 \theta}$$

OBJECTIVE 5

5 Find the Linear Speed of an Object Traveling in Circular Motion



$$\boxed{v = \frac{s}{t}}$$

Linear Speed

$$\boxed{\omega = \frac{\theta}{t}}$$

Angular Speed

$$\boxed{v = r\omega}$$

EXAMPLE Finding Linear Speed

Earth rotates on an axis through its poles. The distance from the axis to a location on Earth 40 north latitude is about 3033.5 miles. Therefore, a location on Earth at 40 north latitude is spinning on a circle of radius 3033.5 miles. Compute the linear speed on the surface of Earth at 40 north latitude.