Section 7.1
Angles and Their Measure
(a) \( \theta \) is in standard position; \( \theta \) is positive

(b) \( \theta \) is in standard position; \( \theta \) is negative
(a) $\theta$ lies in quadrant II

(b) $\theta$ lies in quadrant IV

(c) $\theta$ is a quadrant angle
Degrees

(a) 1 revolution counterclockwise, $360^\circ$

(b) right angle, $\frac{1}{4}$ revolution counter-clockwise, $90^\circ$

(c) straight angle, $\frac{1}{2}$ revolution counter-clockwise, $180^\circ$
EXAMPLE

Drawing an Angle

Draw each angle.

(a) $45^\circ$  (b) $-90^\circ$  (c) $225^\circ$  (d) $405^\circ$
OBJECTIVE 1

1. Convert between Decimals and Degrees, Minutes, Seconds Forms for Angles
1 counterclockwise revolution = 360°

1° = 60′

1′ = 60″
Converting between Degrees, Minutes, Seconds Form and Decimal Form

(a) Convert 45°10'15" to decimal in degrees.
Round the answer to four decimal places.

(b) Convert 21.256° to the D°M'S" form.
Round the answer to the nearest second.
Radians
OBJECTIVE 2

2 Find the Arc Length of a Circle
\[
\frac{\theta}{\theta_1} = \frac{s}{s_1}
\]
Theorem

Arc Length

For a circle of radius $r$, a central angle of $\theta$ radians subtends an arc whose length $s$ is

$$s = r\theta$$
EXAMPLE

Finding the Length of an Arc of a Circle

Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.5 radian.

\[ s = r\theta \]
OBJECTIVE 3

3 Convert from Degrees to Radians and from Radians to Degrees
1 revolution = \(2\pi\) radians

\[ 180^\circ = \pi \text{ radians} \]

1 degree = \(\frac{\pi}{180}\) radian \[ 1 \text{ radian} = \frac{180}{\pi} \text{ degrees} \]
EXAMPLE  Converting from Degrees to Radians

Convert each angle in degrees to radians.

(a) 80  (b) 140  (c) -30  (d) 100

1 degree = \frac{\pi}{180} \text{ radian} \quad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}
EXAMPLE  Converting Radians to Degrees

Convert each angle in radians to degrees.

(a) \( \frac{2\pi}{3} \) radians    (b) \( \frac{5\pi}{6} \) radians    (c) \( \frac{-3\pi}{5} \) radians

(d) \( \frac{8\pi}{3} \) radians    (e) 2 radians

1 degree = \( \frac{\pi}{180} \) radian  \hspace{1cm} 1 radian = \( \frac{180}{\pi} \) degrees
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<th>60°</th>
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EXAMPLE  Finding the Distance between Two Cities

See Figure 13(a). The latitude of a location $L$ is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to $L$. See Figure 13(b). Glasgow, Montana, is due north of Albuquerque, New Mexico. Find the distance between Glasgow ($48^\circ 9'$ north latitude) and Albuquerque ($35^\circ 5'$ north latitude). Assume that the radius of Earth is 3960 miles.
OBJECTIVE 4

4 Find the Area of a Sector of a Circle
The area $A$ of the sector of a circle of radius $r$ formed by a central angle of $\theta$ radians is

$$A = \frac{1}{2}r^2\theta$$
Example

Finding the Area of a Sector of a Circle

Find the area of the sector of a circle of radius 5 feet formed by an angle of 40°. Round the answer to two decimal places.

\[ A = \frac{1}{2} r^2 \theta \]
OBJECTIVE 5

Find the Linear Speed of an Object Traveling in Circular Motion
Linear Speed

\[ v = \frac{s}{t} \]

Angular Speed

\[ \omega = \frac{\theta}{t} \]

\[ v = r\omega \]
Earth rotates on an axis through its poles. The distance from the axis to a location on Earth at 40° north latitude is about 3033.5 miles. Therefore, a location on Earth at 40° north latitude is spinning on a circle of radius 3033.5 miles. Compute the linear speed on the surface of Earth at 40° north latitude.