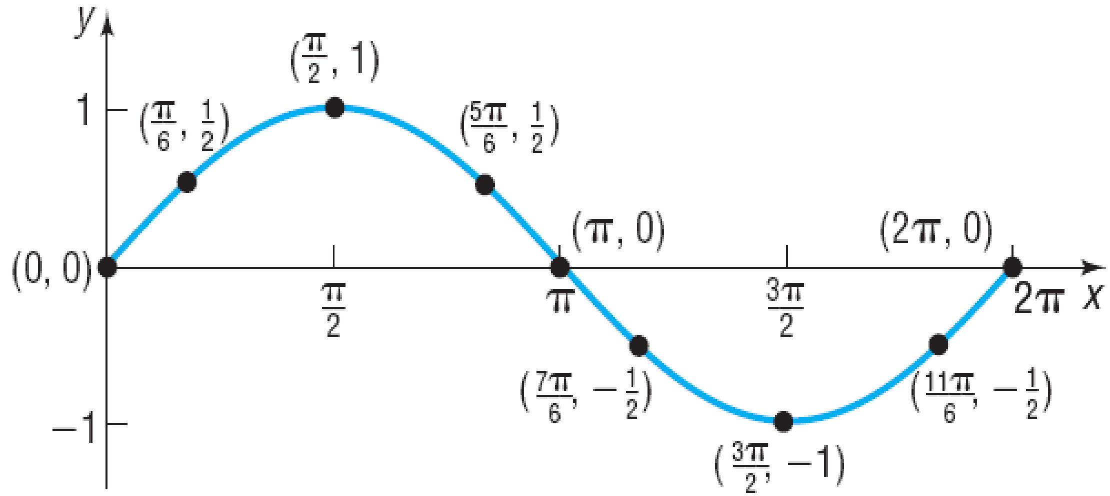
Section 7.6

Graphs of the Sine and Cosine Functions

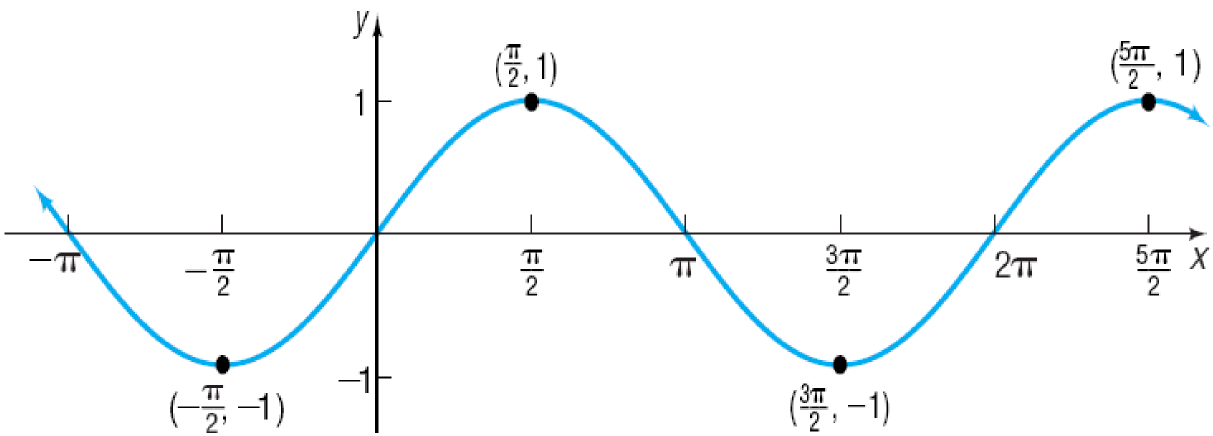
$y = f(x) = \sin x$	$y = f(x) = \cos x$	$y = f(x) = \tan x$
$y = f(x) = \csc x$	$y = f(x) = \sec x$	$y = f(x) = \cot x$

The Graph of the Sine Function $y = \sin x$

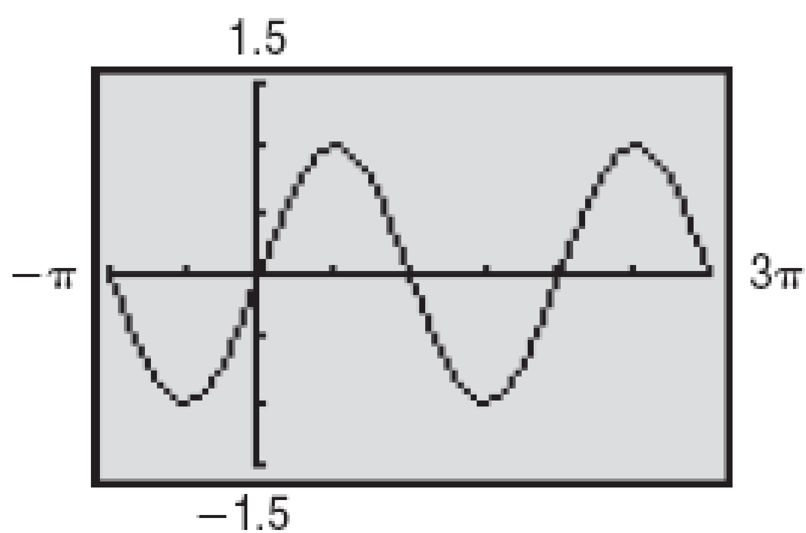
x	$y = \sin x$	(x, y)
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$



$y = \sin x, 0 \leq x \leq 2\pi$



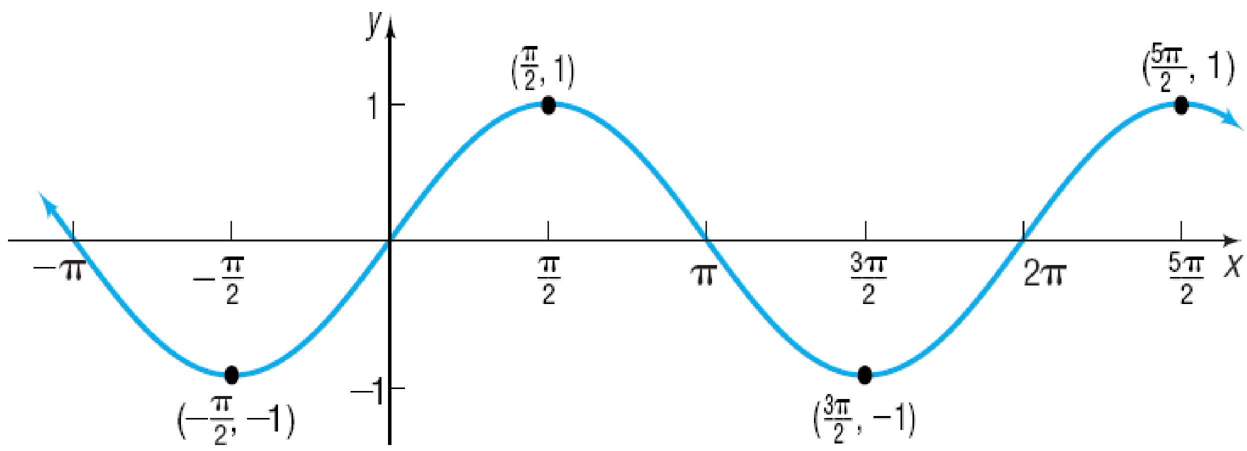
$$y = \sin x, -\infty < x < \infty$$



Properties of the Sine Function $y = \sin x$

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$;

   the minimum value is -1 and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.



OBJECTIVE 1

1 Graph Functions of the Form $y = A\sin(\omega x)$ Using Transformations

EXAMPLE

Graphing Functions of the Form $y = A \sin(\omega x)$ Using Transformations

Graph $y = 5 \sin x$ using transformations. Use the graph to determine the domain and the range of $y = 5 \sin x$.

EXAMPLE

Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

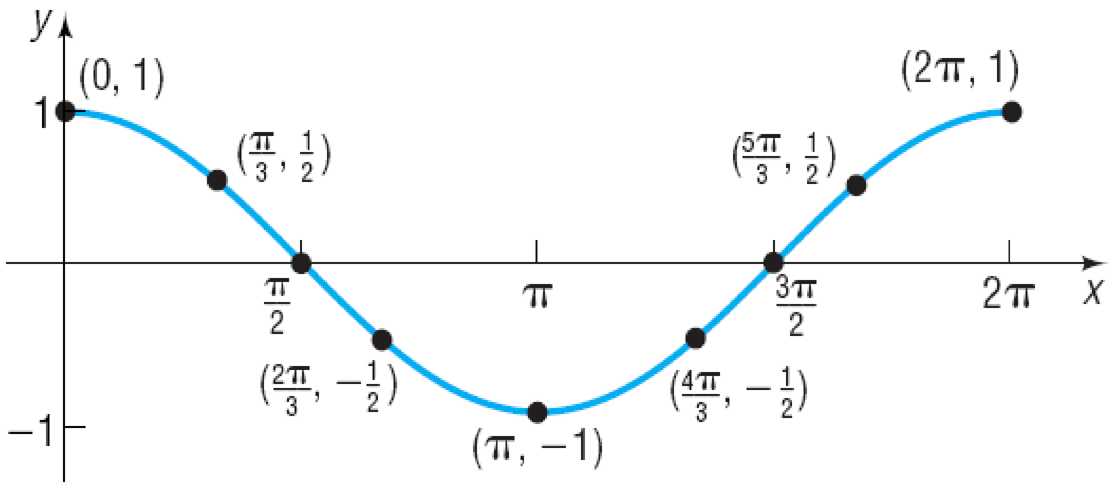
Graph $y = \frac{1}{2}\sin(-\pi x)$ using transformations.

Use the graph to determine the domain and the range of

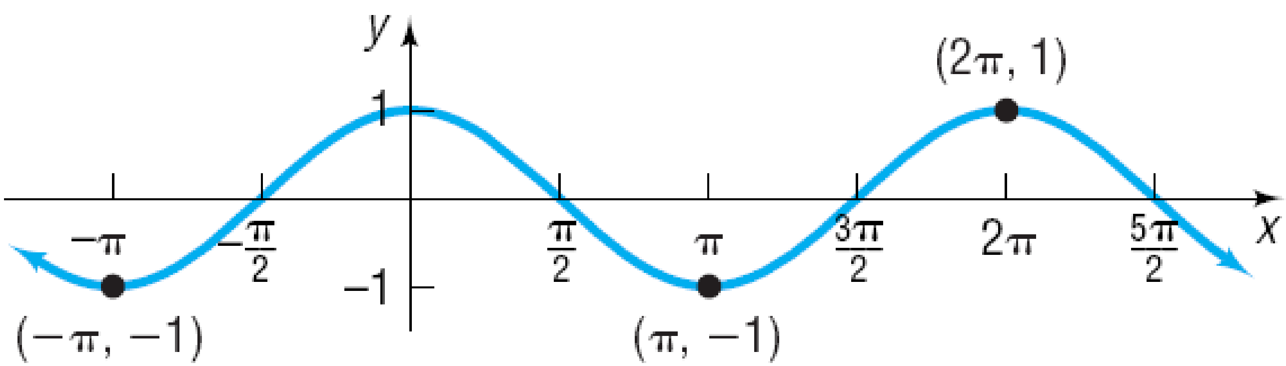
$$y = \frac{1}{2}\sin(-\pi x)$$

The Graph of the Cosine Function

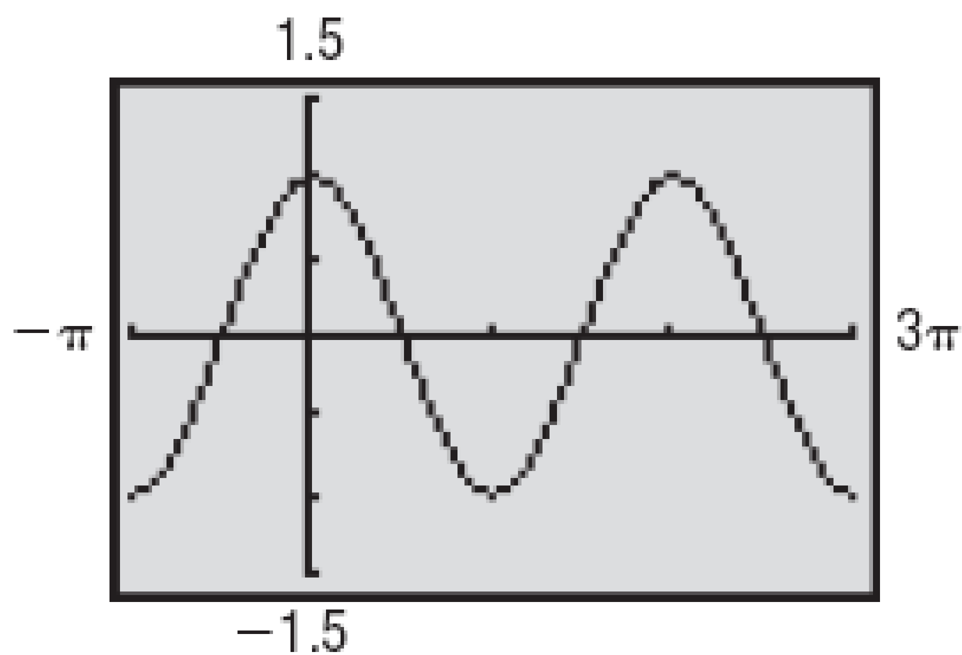
x	$y = \cos x$	(x, y)
0	1	$(0, 1)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$



$$y = \cos x,\ 0 \le x \le 2\pi$$

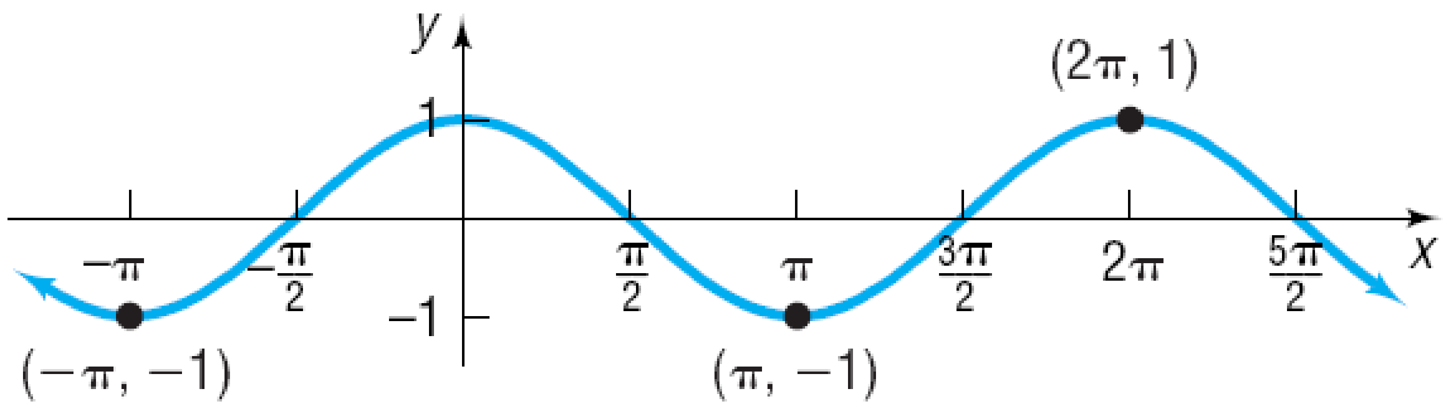


$$y = \cos x, -\infty < x < \infty$$



Properties of the Cosine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.



OBJECTIVE 2

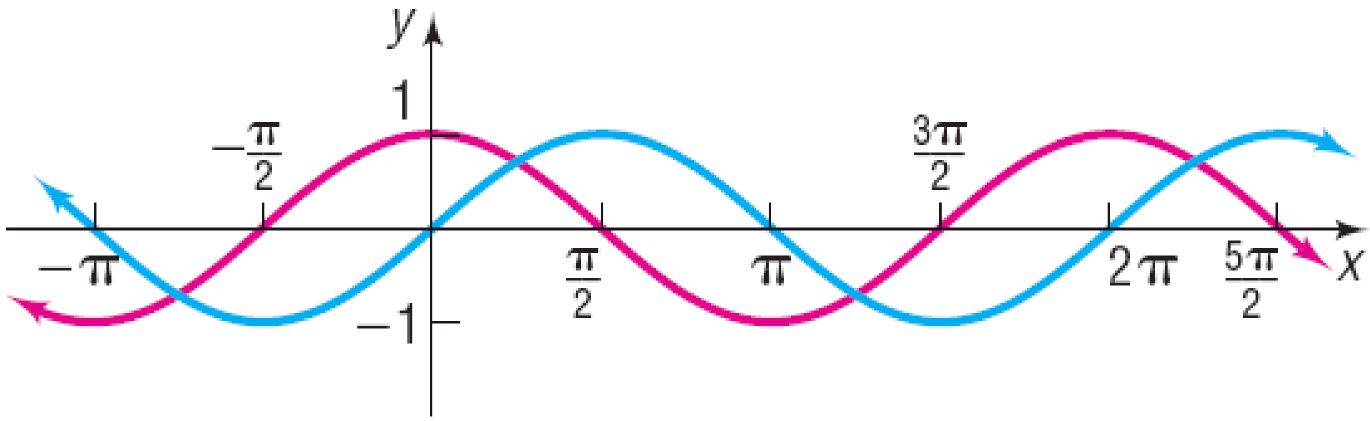
2 Graph Functions of the Form $y = A\cos(\omega x)$ Using Transformations

EXAMPLE

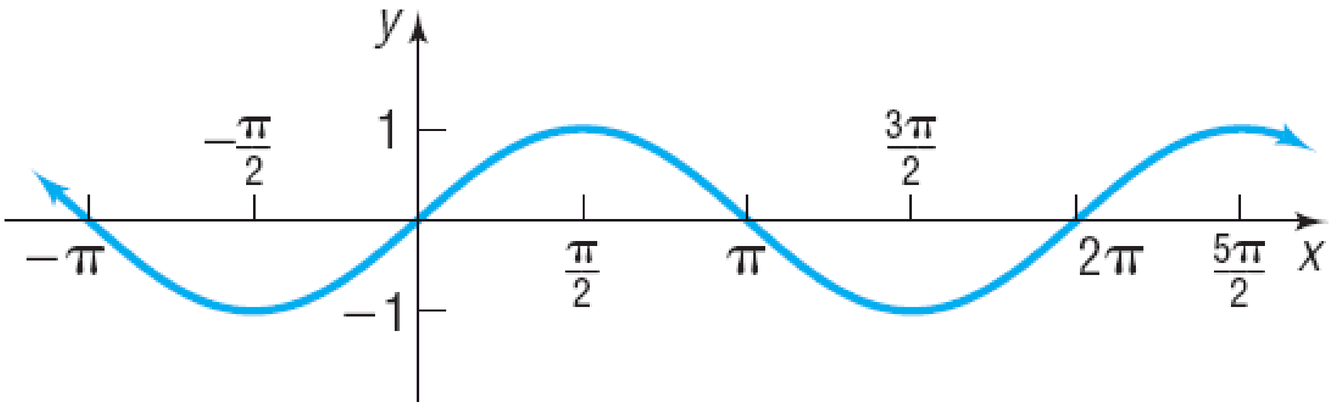
Graphing Functions of the Form $y = A\cos(\omega x)$ Using Transformations

Graph $y = - \ \cos(2x)$ using transformations. Use the graph to determine the domain and the range of $y = - \ \cos(2x)$.

Sinusoidal Graphs



(a) $y = \cos x$ $y = \cos\left(x - \frac{\pi}{2}\right)$



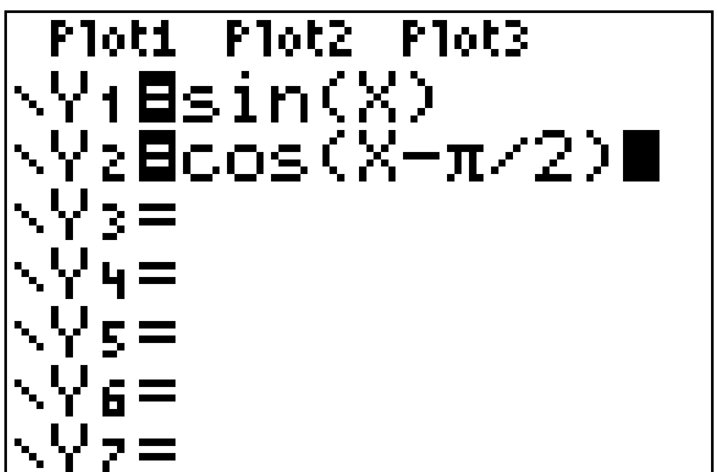
(b) $y = \sin x$

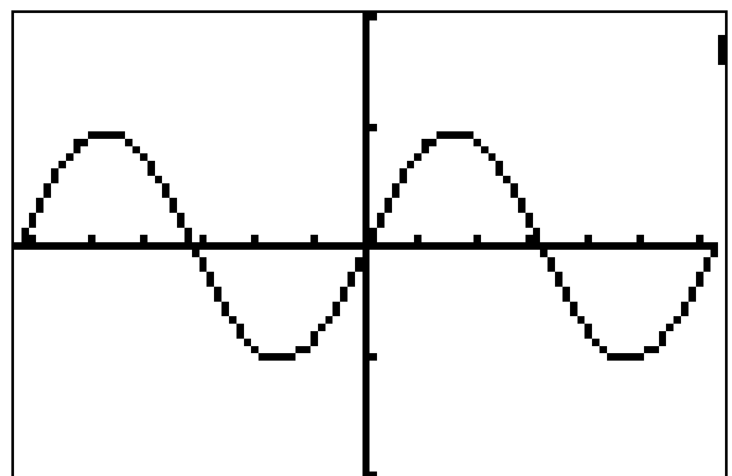
$$\sin x = \cos\left(x - \frac{\pi}{2}\right)$$

Seeing the Concept

Graph $Y_1 = \sin x$ and $Y_2 = \cos\left(x - \frac{\pi}{2}\right)$.

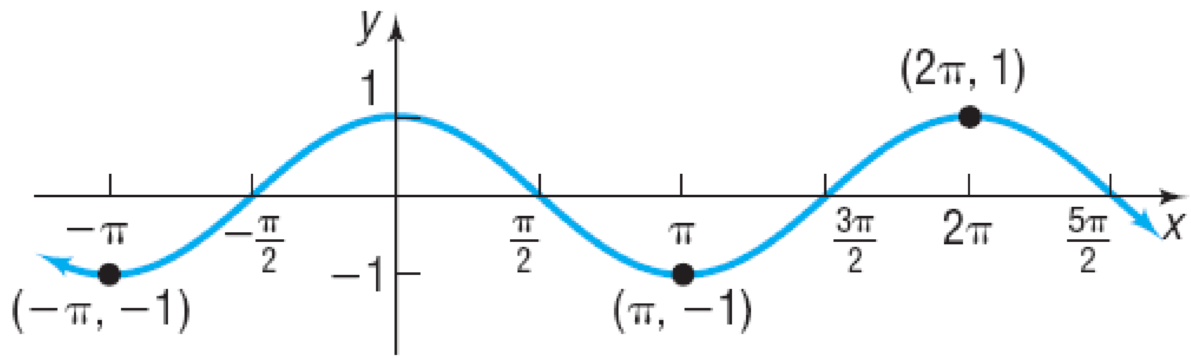
How many graphs do you see?





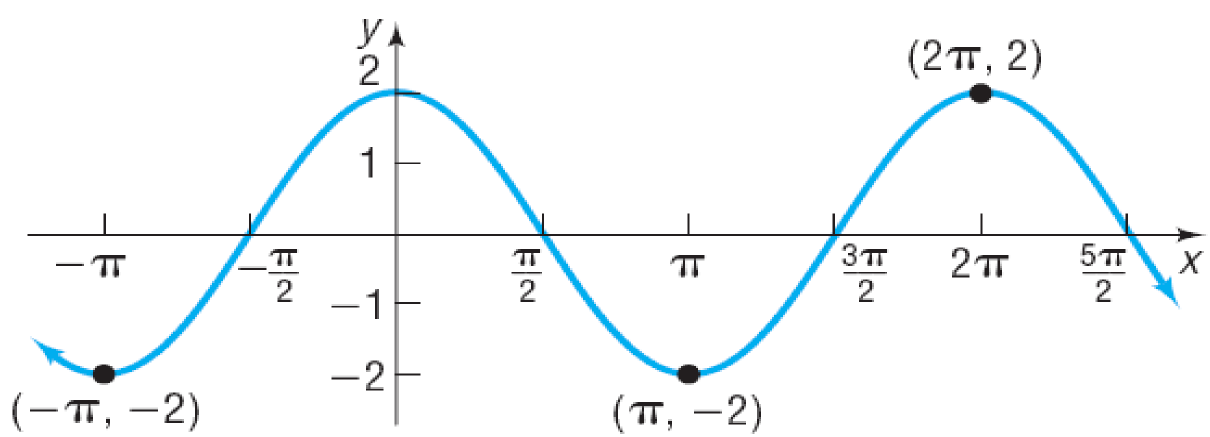
OBJECTIVE 3

3 Determine the Amplitude and Period of Sinusoidal Functions



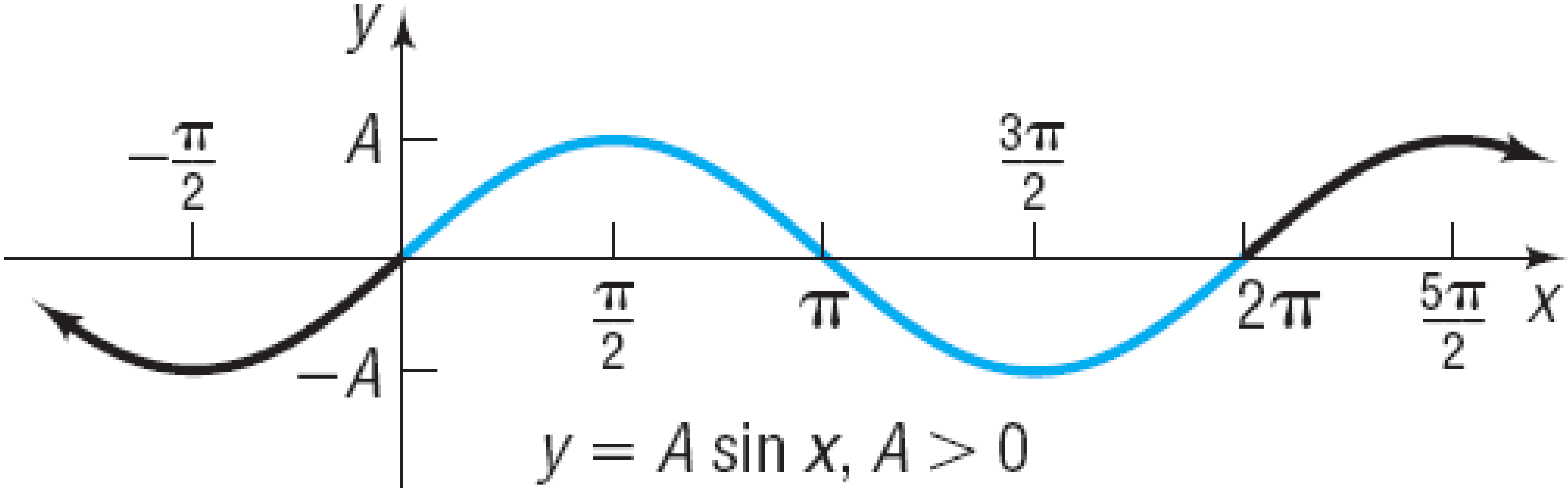
$y = \cos x$

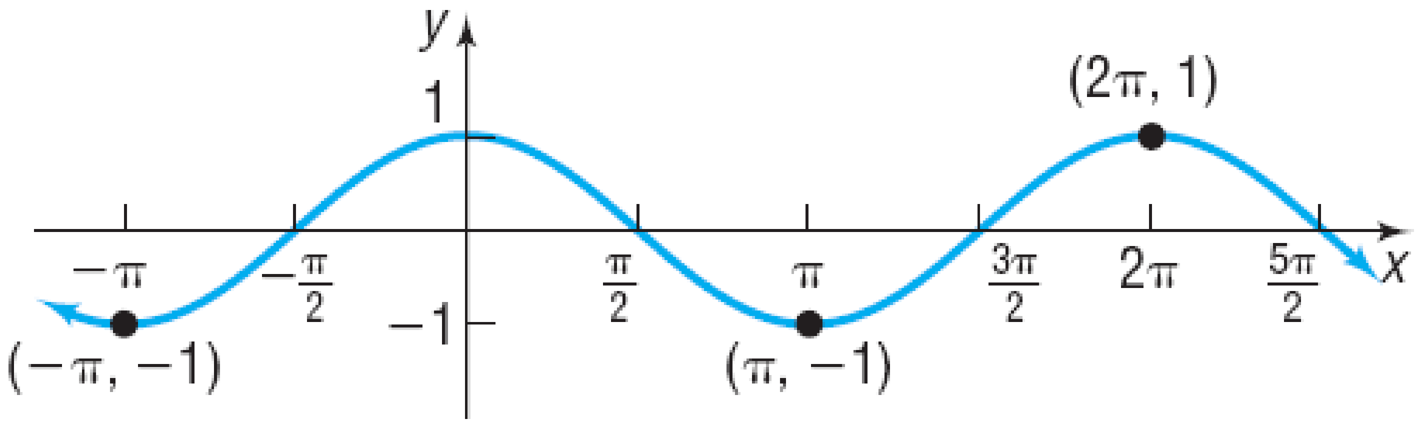
Multiply by 2;
Vertical stretch
by a factor of 2



$y = 2 \cos x$

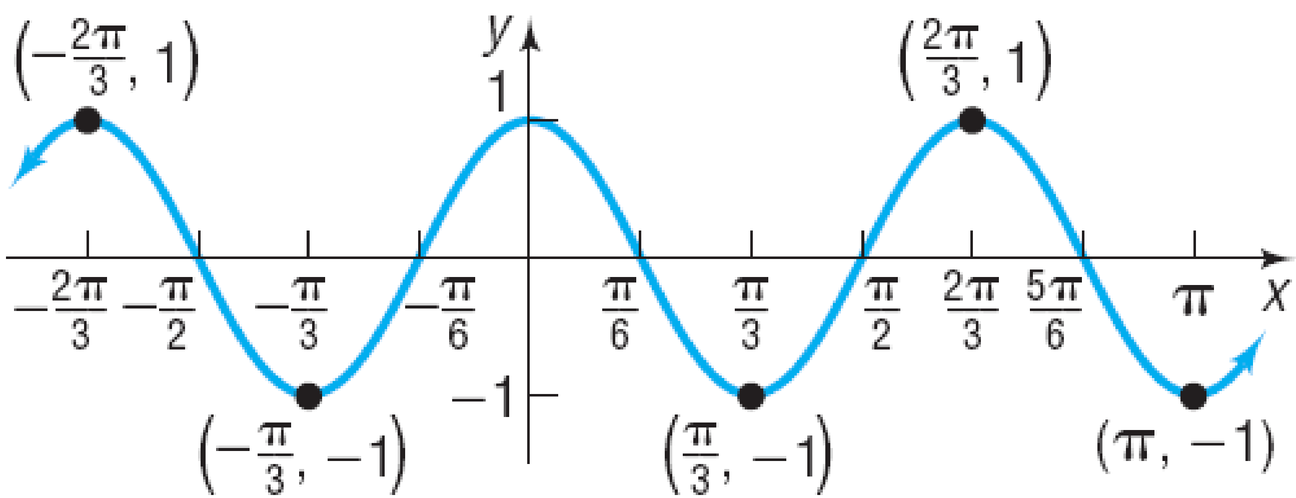
Amplitude



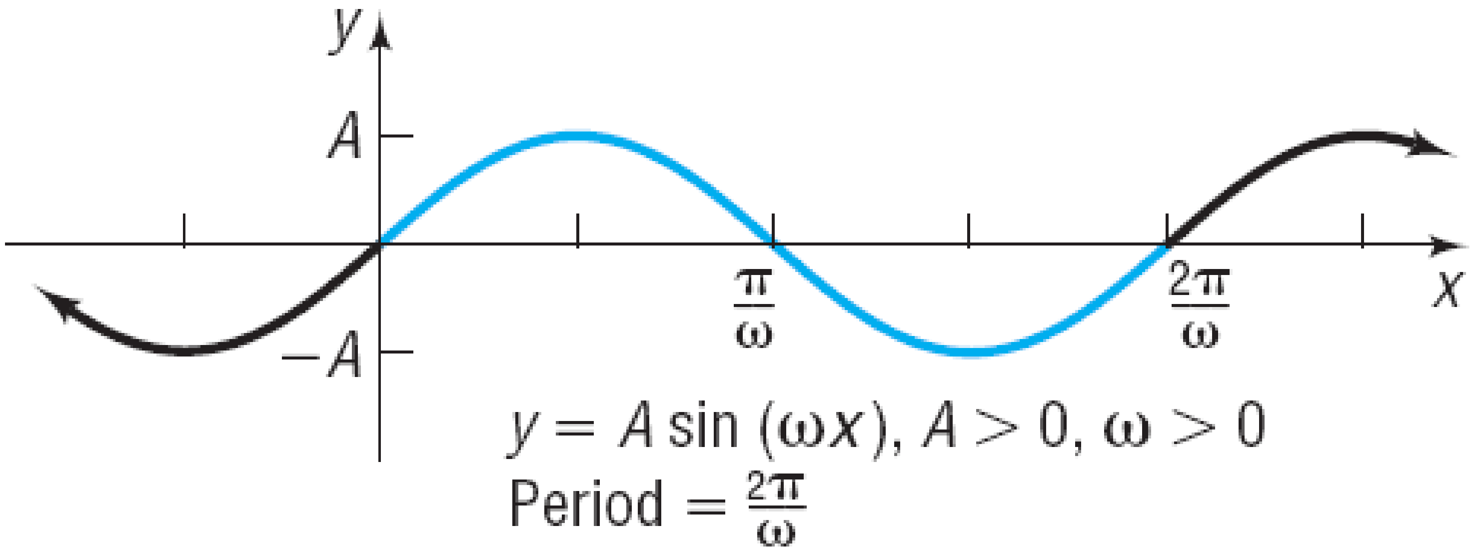


$y = \cos x$

Replace x by $3x$;
Horizontal compression
by a factor of $\frac{1}{3}$



$y = \cos(3x)$



Theorem

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}$$

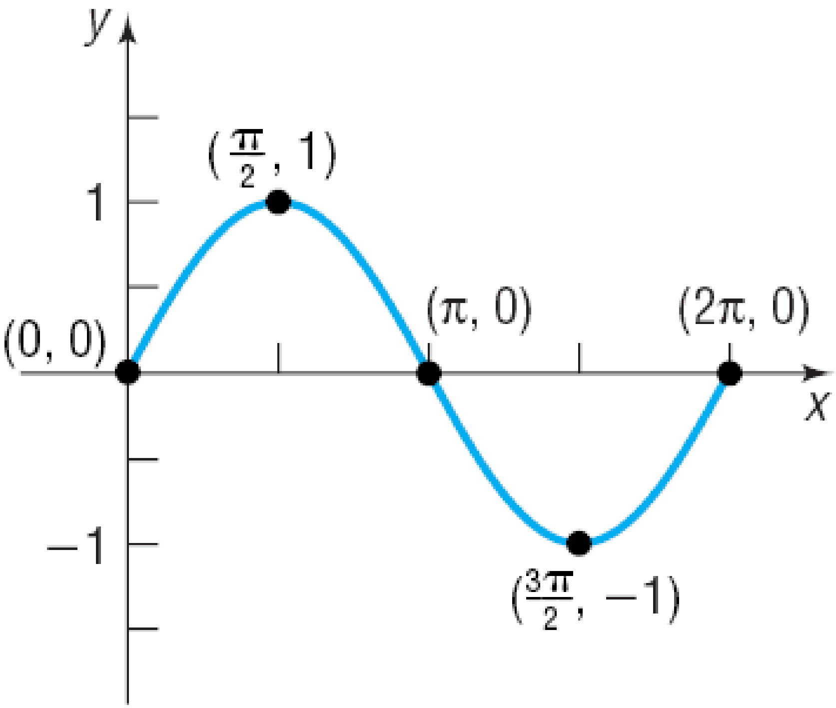
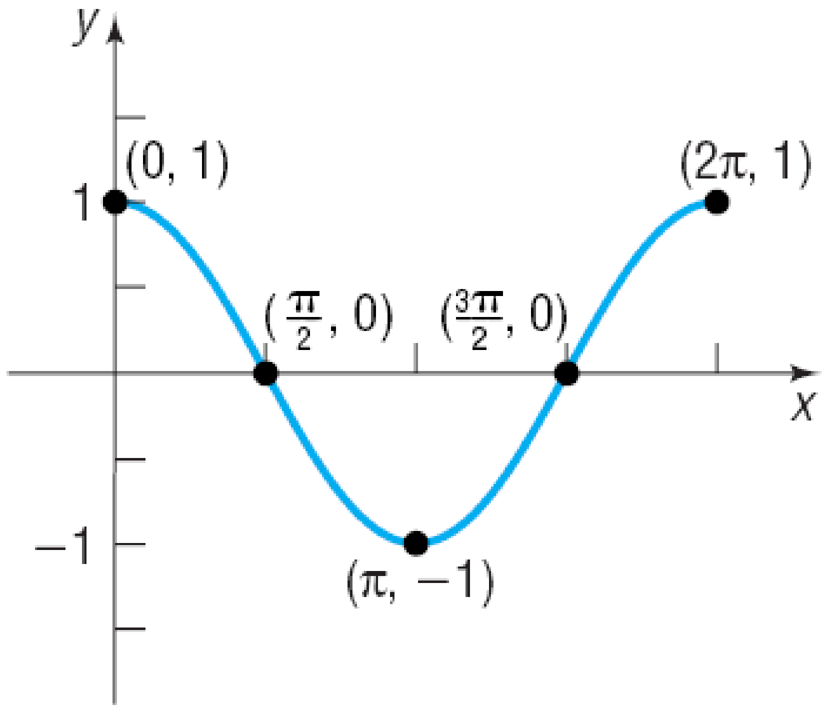
EXAMPLE

Finding the Amplitude and Period of a Sinusoidal Function

Determine the amplitude and period of $y = -4\cos(3x)$

OBJECTIVE 4

4 Graph Sinusoidal Functions Using Key Points


y
1
−1
x
(0, 0)
(π/2, 1)
(π, 0)
(2π, 0)
(3π/2, −1)

y
1
−1
x
(0, 1)
(2π, 1)
(π/2, 0)
(3π/2, 0)
(π, −1)

EXAMPLE

How to Graph a Sinusoidal Function Using Key Points

Graph $y = 4 \sin (2x)$ using key points.

SUMMARY Steps for Graphing a Sinusoidal Function of the Form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ Using Key Points

Step 1: Determine the amplitude and period of the sinusoidal function.

Step 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

Step 3: Use the endpoints of these subintervals to obtain five key points on the graph.

Step 4: Plot the five key points with a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.

EXAMPLE

Graphing a Sinusoidal Function Using Key Points

Graph $y = -5\cos\left(-\frac{\pi}{2}x\right)$ using key points.

EXAMPLE

Graphing a Sinusoidal Function Using Key Points

Graph $y = 3\cos\left(\frac{\pi}{2}x\right) + 1$ using key points.

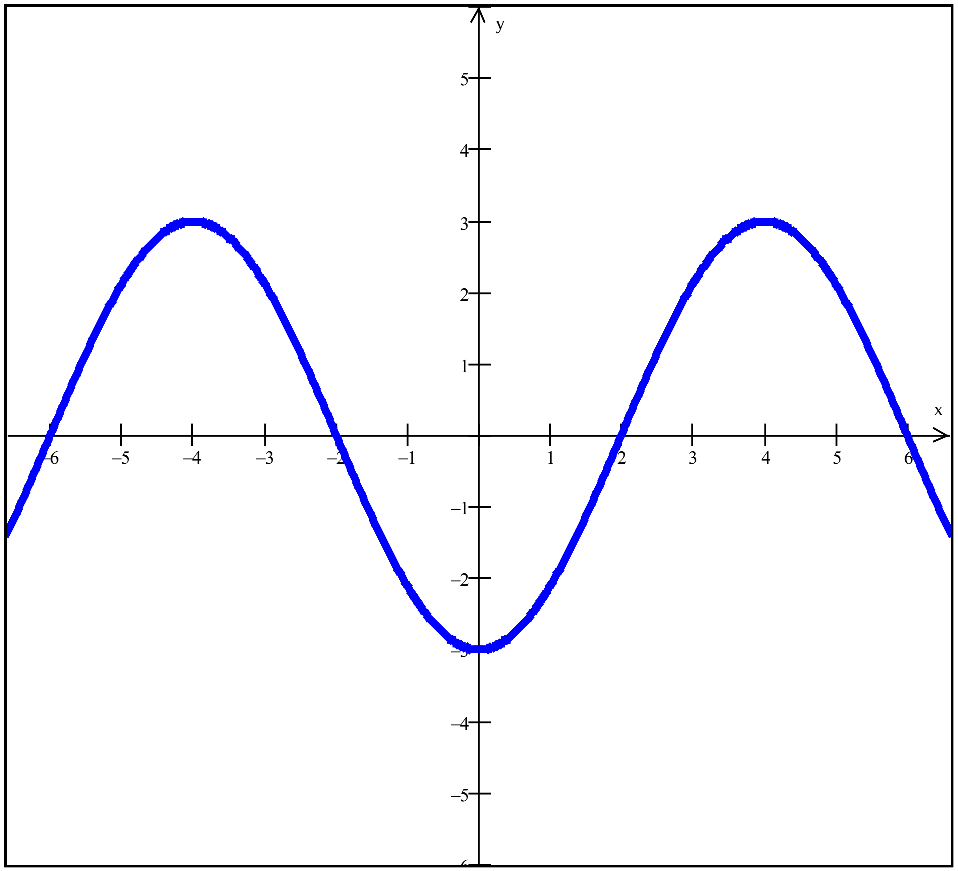
OBJECTIVE 5

5 Find an Equation for a Sinusoidal Graph

EXAMPLE

Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown



EXAMPLE

Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown