# Section 7.8

# Phase Shift;

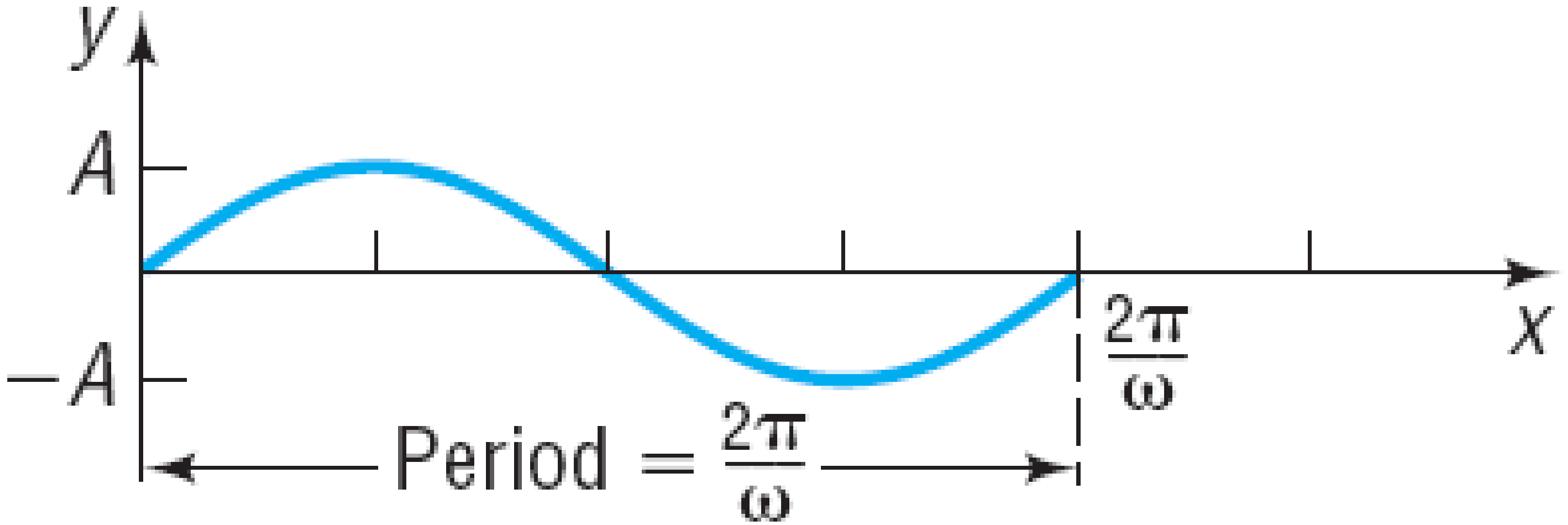
# Building Sinusoidal Models

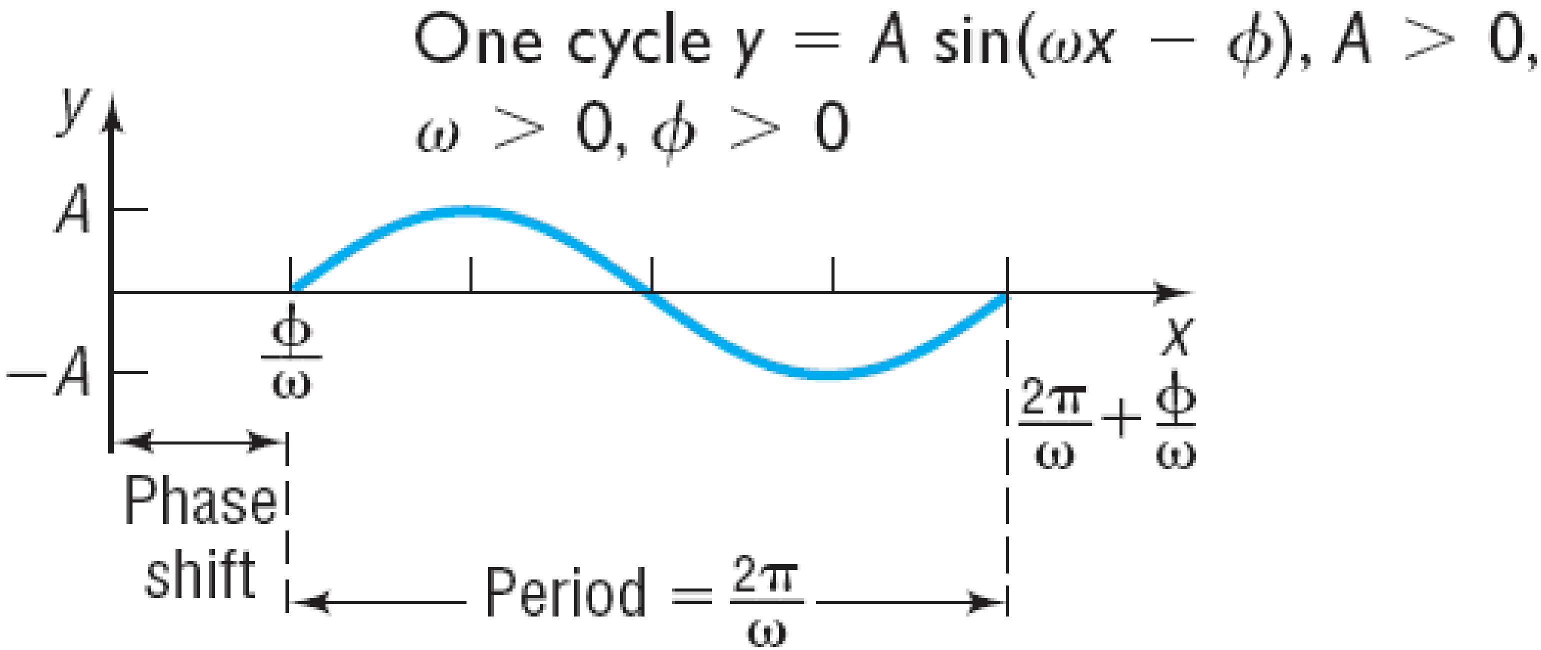
# OBJECTIVE 1

1 Graph Sinusoidal Functions of the Form $y = A\sin(\omega x - \phi) + B$

One cycle

$y = A\sin(\omega x),\ A > 0,\ \omega > 0$






For the graphs of $y = A\sin(\omega x - \phi)$ or $y = A\cos(\omega x - \phi)$, $\omega > 0$,

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega} \qquad \text{Phase shift} = \frac{\phi}{\omega}$$

The phase shift is to the left if $\phi < 0$ and to the right if $\phi > 0$.

**EXAMPLE**

## Finding the Amplitude, Period, and Phase Shift of a Sinusoidal Function and Graphing It

Find the amplitude, period and phase shift of

$y = 5\sin\left(2x + 5\right)$ and graph the function.

EXAMPLE

## Finding the Amplitude, Period, and Phase Shift of a Sinusoidal Function and Graphing It

Find the amplitude, period and phase shift of

$y = -3\cos(-4x + \pi)$ and graph the function.

## SUMMARY Steps for Graphing Sinusoidal Functions $y = A\sin(\omega x - \phi) + B$ or $y = A\cos(\omega x - \phi) + B$

**STEP 1:** Determine the amplitude $|A|$ and period $T = \frac{2\pi}{\omega}$.

**STEP 2:** Determine the starting point of one cycle of the graph, $\frac{\phi}{\omega}$. Determine the ending point of one cycle of the graph, $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$. Divide the interval $\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + \frac{2\pi}{\omega}\right]$ into four subintervals, each of length $\frac{2\pi}{\omega} \div 4$.

**STEP 3:** Use the endpoints of the subintervals to find the five key points on the graph.

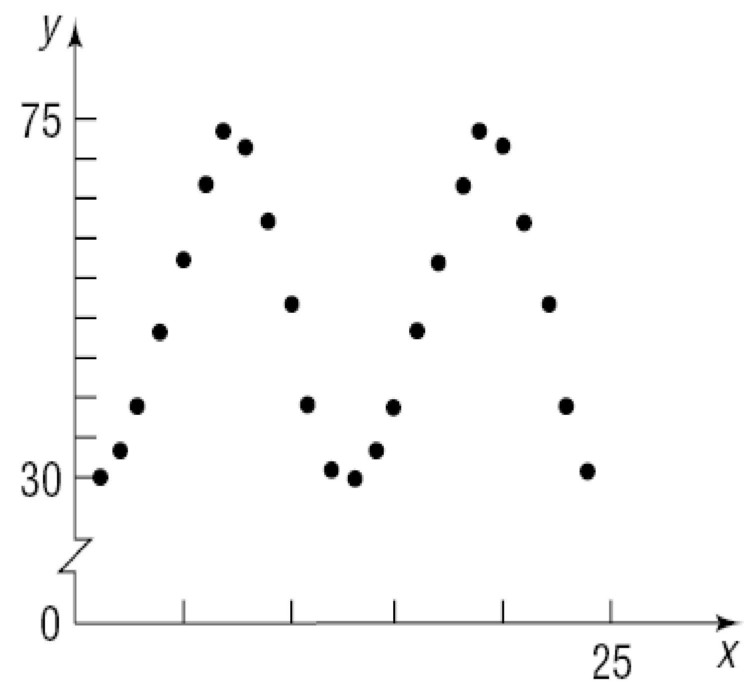
**STEP 4:** Plot the five key points with a sinusoidal graph to obtain one cycle of the graph. Extend the graph in each direction to make it complete.

**STEP 5:** If $B \neq 0$, apply a vertical shift.

# OBJECTIVE 2

2 Build Sinusoidal Models from Data

| Month, $x$ | Average Monthly Temperature, °F |
|---|---|
| January, 1 | 29.7 |
| February, 2 | 33.4 |
| March, 3 | 39.0 |
| April, 4 | 48.2 |
| May, 5 | 57.2 |
| June, 6 | 66.9 |
| July, 7 | 73.5 |
| August, 8 | 71.4 |
| September, 9 | 62.3 |
| October, 10 | 51.4 |
| November, 11 | 39.0 |
| December, 12 | 31.0 |



**EXAMPLE** **Finding a Sinusoidal Function from Temperature Data**

Fit a sine function to the data in Table

| Month, $x$ | Average Monthly Temperature, °F |
|---|---|
| January, 1 | 29.7 |
| February, 2 | 33.4 |
| March, 3 | 39.0 |
| April, 4 | 48.2 |
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| November, 11 | 39.0 |
| December, 12 | 31.0 |

**STEP 1:** Determine $A$, the amplitude of the function.

$$\text{Amplitude} = \frac{\text{largest data value} - \text{smallest data value}}{2}$$

**EXAMPLE** **Finding a Sinusoidal Function from Temperature Data**

Fit a sine function to the data in Table

| Month, $x$ | Average Monthly Temperature, °F |
|---|---|
| January, 1 | 29.7 |
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**Step 2:** Determine $B$, the vertical shift of the function.

$$\text{Vertical shift} = \frac{\text{largest data value} + \text{smallest data value}}{2}$$

EXAMPLE

## Finding a Sinusoidal Function from Temperature Data

Fit a sine function to the data in Table

| Month, $x$ | Average Monthly Temperature, °F |
|---|---|
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**STEP 3:** Determine $\omega$. Since the period $T$, the time it takes for the data to repeat, is $T = \frac{2\pi}{\omega}$, we have

$$\omega = \frac{2\pi}{T}$$

**EXAMPLE** **Finding a Sinusoidal Function from Temperature Data**

Fit a sine function to the data in Table

| Month, $x$ | Average Monthly Temperature, °F |
|---|---|
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**STEP 4:** Determine the horizontal shift of the function by using the period of the data. Divide the period into four subintervals of equal length. Determine the $x$-coordinate for the maximum of the sine function and the $x$-coordinate for the maximum value of the data. Use this information to determine the value of the phase shift, $\frac{\phi}{\omega}$.

## Steps for Fitting Data to a Sine Function $y = A\sin(\omega x - \phi) + B$

**Step 1:** Determine $A$, the amplitude of the function.

$$\text{Amplitude} = \frac{\text{largest data value} - \text{smallest data value}}{2}$$

**Step 2:** Determine $B$, the vertical shift of the function.

$$\text{Vertical shift} = \frac{\text{largest data value} + \text{smallest data value}}{2}$$

**Step 3:** Determine $\omega$. Since the period $T$, the time it takes for the data to repeat, is $T = \frac{2\pi}{\omega}$, we have

$$\omega = \frac{2\pi}{T}$$

**Step 4:** Determine the horizontal shift of the function by using the period of the data. Divide the period into four subintervals of equal length. Determine the $x$-coordinate for the maximum of the sine function and the $x$-coordinate for the maximum value of the data. Use this information to determine the value of the phase shift, $\frac{\phi}{\omega}$.

EXAMPLE

# Finding a Sinusoidal Function for Hours of Daylight

According to the *Old Farmer's Almanac*, the number of hours of sunlight in Boston on the summer solstice is 15.30 and the number of hours of sunlight on the winter solstice is 9.08.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac* and compare it to the results found in part (b).

## EXAMPLE Finding the Sine Function of Best Fit

Use a graphing utility to find the sine function of best fit that models the data in the Table. Graph this function with the scatter diagram of data.

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