Section 8.4
Sum and Difference Formulas
Theorem

Sum and Difference Formulas for Cosines

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]

\[
\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta
\]
Theorem

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\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
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\]
OBJECTIVE 1

1. Use Sum and Difference Formulas to Find Exact Values
Using the Sum Formula to Find Exact Values

Find the exact value of $\cos 15^\circ$. 
EXAMPLE

Using the Difference Formula to Find Exact Values

Find the exact value of \( \cos \frac{7\pi}{12} \).
OBJECTIVE 2

2 Use Sum and Difference Formulas to Establish Identities
\[
\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \\
\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta
\]

— Seeing the Concept —

Graph \( Y_1 = \cos\left(\frac{\pi}{2} - \theta\right) \) and \( Y_2 = \sin \theta \) on the same screen. Does this demonstrate the result 3(a)? How would you demonstrate the result 3(b)?
Theorem

Sum and Difference Formulas for Sines

\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \\
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta
\]
Using the Sum Formula to Find Exact Values

Find the exact value of $\sin \frac{19\pi}{12}$. 
Using the Difference Formula to Find Exact Values

Find the exact value of \( \cos 40^\circ \cos 80^\circ - \sin 40^\circ \sin 80^\circ \).
EXAMPLE  Finding Exact Values

If it is known that $\sin \alpha = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and that $\sin \beta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$, $\frac{3\pi}{2} < \beta < 2\pi$, find the exact value of

(a) $\cos \alpha$ \hspace{1cm} (b) $\cos \beta$ \hspace{1cm} (c) $\cos(\alpha + \beta)$ \hspace{1cm} (d) $\sin(\alpha + \beta)$
EXAMPLE Establishing an Identity

Establish the identity: \[ \cos \left( \frac{\pi}{2} + \theta \right) = -\sin \theta \]
Theorem

Sum and Difference Formulas for Tangents

\[
\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
\]

\[
\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}
\]
EXAMPLE  Establishing an Identity

Prove the identity: \[ \tan(2\pi - \theta) = -\tan \theta \]
EXAMPLE  Establishing an Identity

Prove the identity: \( \tan\left(\frac{\pi}{4} + \theta\right) = \cot\left(\frac{\pi}{4} - \theta\right) \)
Objective 3

3. Use Sum and Difference Formulas Involving Inverse Trigonometric Functions
EXAMPLE

Finding the Exact Value of an Expression Involving Inverse Trigonometric Functions

Find the exact value of: \( \cos \left( \sin^{-1} \frac{2}{3} + \tan^{-1} \left( -\frac{3}{4} \right) \right) \)
EXAMPLE

Writing a Trigonometric Expression as an Algebraic Expression

Write $\cos \left( \cos^{-1} u - \sin^{-1} v \right)$ as an algebraic expression containing $u$ and $v$ (that is, without any trigonometric functions). Give the restrictions on $u$ and $v$. 
Summary

Sum and Difference Formulas

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \\
\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
\]

\[
\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \\
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\
\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}
\]