Section 8.4

Sum and Difference Formulas

Theorem

Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

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Sum and Difference Formulas for Cosines

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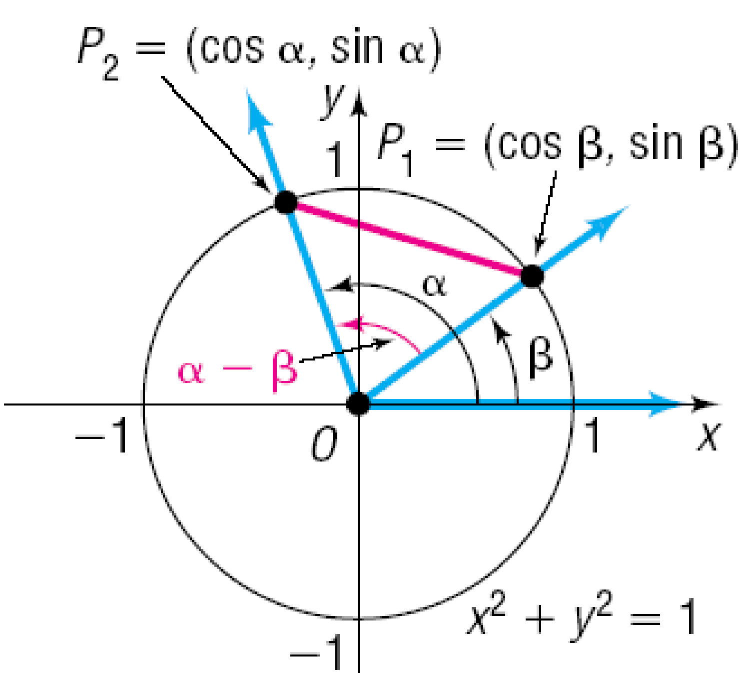

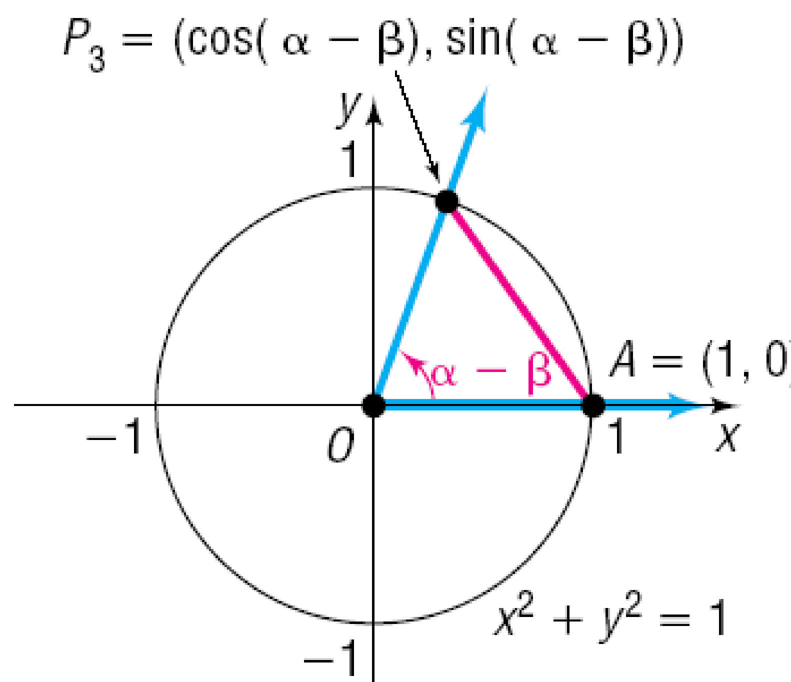

$$d(A, P_3) = d(P_1, P_2)$$

OBJECTIVE 1

1 Use Sum and Difference Formulas to Find Exact Values

EXAMPLE

Using the Sum Formula to Find Exact Values

Find the exact value of $\cos 15°$.

EXAMPLE

Using the Difference Formula to Find Exact Values

Find the exact value of $\cos\dfrac{7\pi}{12}$.

OBJECTIVE 2

2 Use Sum and Difference Formulas to Establish Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

Seeing the Concept

Graph $Y_1 = \cos\left(\frac{\pi}{2} - \theta\right)$ and $Y_2 = \sin\theta$ on the same screen. Does this demonstrate the result 3(a)? How would you demonstrate the result 3(b)?

Theorem

Sum and Difference Formulas for Sines

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

EXAMPLE

Using the Sum Formula to Find Exact Values

Find the exact value of $\sin \dfrac{19\pi}{12}$.

EXAMPLE

Using the Difference Formula to Find Exact Values

Find the exact value of $\cos 40° \cos 80° - \sin 40° \sin 80°$.

EXAMPLE Finding Exact Values

If it is known that $\sin\alpha = \frac{3}{5}, \ \frac{\pi}{2} < \alpha < \pi,$ and that

$\sin\beta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}, \frac{3\pi}{2} < \beta < 2\pi,$ find the exact value of

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos(\alpha + \beta)$ (d) $\sin(\alpha + \beta)$

EXAMPLE **Establishing an Identity**

Establish the identity: $$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$$

Theorem

Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

EXAMPLE Establishing an Identity

Prove the identity: $\tan(2\pi - \theta) = -\tan\theta$

EXAMPLE Establishing an Identity

Prove the identity: $\tan\left(\frac{\pi}{4}+\theta\right)=\cot\left(\frac{\pi}{4}-\theta\right)$

OBJECTIVE 3

3 Use Sum and Difference Formulas Involving Inverse Trigonometric Functions

EXAMPLE

Finding the Exact Value of an Expression Involving Inverse Trigonometric Functions

Find the exact value of: $\cos\left(\sin^{-1}\frac{2}{3}+\tan^{-1}\left(-\frac{3}{4}\right)\right)$

EXAMPLE

Writing a Trigonometric Expression as an Algebraic Expression

Write $\cos\left(\cos^{-1} u - \sin^{-1} v\right)$ as an algebraic expession containing u and v (that is, without any trigonometric functions).

Give the restrictions on u and v.

Summary

Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$