Section 8.5

Double-angle and Half-angle Formulas

Theorem

Double-angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

OBJECTIVE 1

1 Use Double-angle Formulas to Find Exact Values

EXAMPLE

Finding Exact Values Using the Double-angle Formula

If $\cos\theta = -\dfrac{2}{5}, \pi < \theta < \dfrac{3\pi}{2},$ find the exact value of:

(a) $\sin(2\theta)$

(b) $\cos(2\theta)$

OBJECTIVE 2

2 Use Double-angle Formulas to Establish Identities

EXAMPLE Establishing Identities

(a) Develop a formula for $\tan(2\theta)$ in terms of $\tan\theta$.

(b) Develop a formula for $\sin(3\theta)$ in terms of $\sin\theta$ and $\cos\theta$.

$$\boxed{\sin^2\theta = \frac{1 - \cos(2\theta)}{2}}$$

$$\boxed{\cos^2\theta = \frac{1 + \cos(2\theta)}{2}}$$

$$\boxed{\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}}$$

EXAMPLE Establishing an Identity

Write an equivalent expression for $\cos^4 \theta$ that does not involve any powers of sine or cosine greater than 1.

EXAMPLE

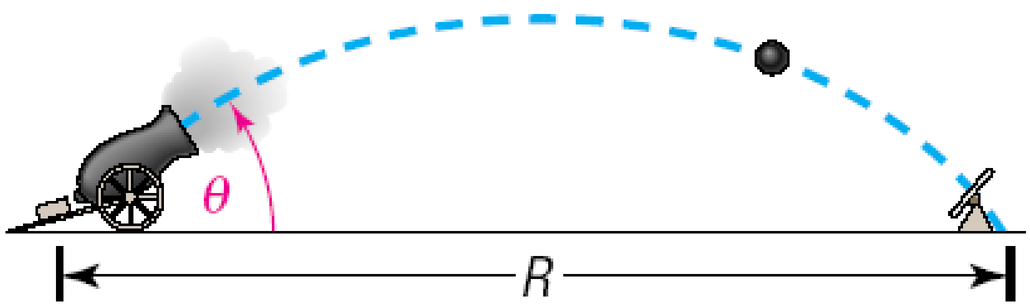
Projectile Motion

An object is propelled upward at an angle θ to the horizontal with an initial velocity of v_0 feet per second. See Figure 28. If air resistance is ignored, the **range** R, the horizontal distance that the object travels, is given by

$$R = \frac{1}{16} v_0^2 \sin \theta \cos \theta$$

(a) Show that $R = \dfrac{1}{32} v_0^2 \sin(2\theta)$.

(b) Find the angle θ for which R is a maximum.



OBJECTIVE 3

3 Use Half-angle Formulas to Find Exact Values

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} \qquad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

Theorem

Half-angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

EXAMPLE

Finding Exact Values Using Half-angle Formulas

Use a Half-angle Formula to find the exact value of:

(a) $\sin 22.5$ (b) $\cos \dfrac{5\pi}{12}$

EXAMPLE

Finding Exact Values Using Half-angle Formulas

If $\tan\alpha = -\frac{1}{5}, \frac{\pi}{2} < \alpha < \pi,$ find the exact value of:

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{2}$ (c) $\tan\frac{\alpha}{2}$

Half-angle Formulas for $\tan \frac{\alpha}{2}$

$$\boxed{\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}}$$