Section 9.3

The Law of Cosines

Case 3: Two sides and the included angle are known (SAS).

Case 4: Three sides are known (SSS).

Theorem

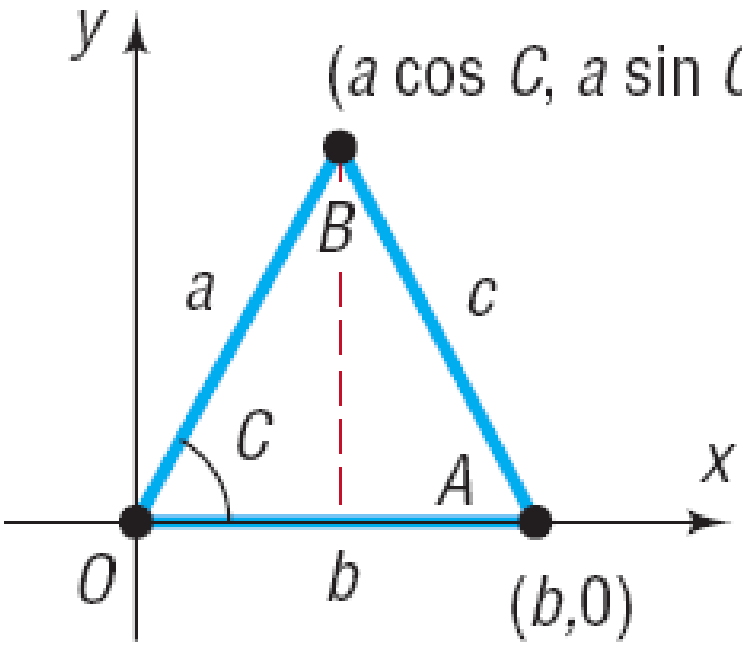
Law of Cosines

For a triangle with sides a, b, c and opposite angles A, B, C, respectively,

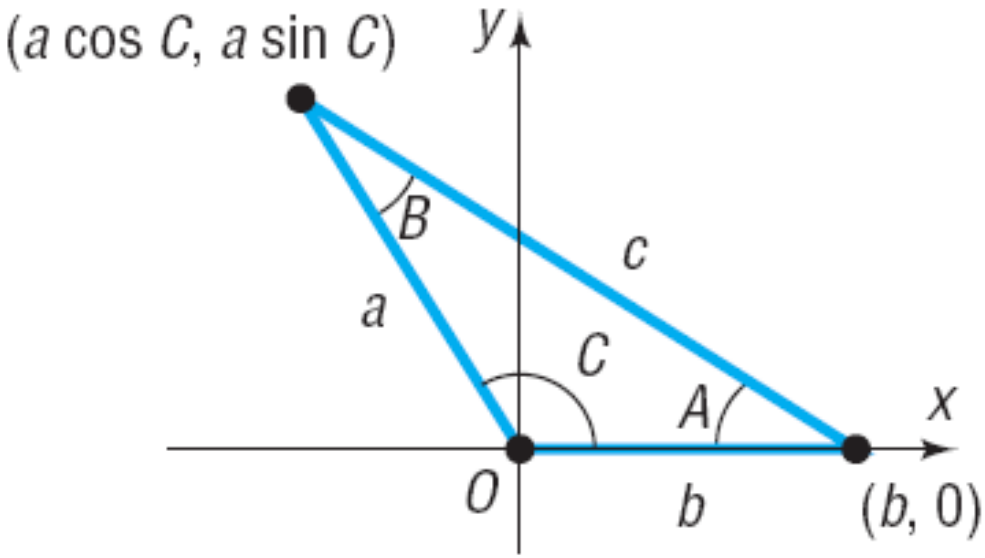
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$



(a) Angle C is acute



(b) Angle C is obtuse

Theorem

Law of Cosines

The square of one side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

OBJECTIVE 1

1 Solve SAS Triangles

EXAMPLE

Using the Law of Cosines to Solve a SAS Triangle

Solve the triangle: $b = 5$, $c = 8$, $A = 35$

OBJECTIVE 2

2 Solve SSS Triangles

EXAMPLE

Using the Law of Cosines to Solve a SSS Triangle

Solve the triangle: $a = 6$, $b = 8$, $c = 5$

OBJECTIVE 3

3 Solve Applied Problems

EXAMPLE Correcting a Navigational Error

A motorized sailboat leaves Naples, Florida, bound for Key West, 150 miles away. Maintaining a constant speed of 15 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds, after 4 hours, that the sailboat is off course by 20°.

(a) How far is the sailboat from Key West at this time?

(b) Through what angle should the sailboat turn to correct its course?

(c) How much time has been added to the trip because of this? (Assume that the speed remains at 15 miles per hour.)