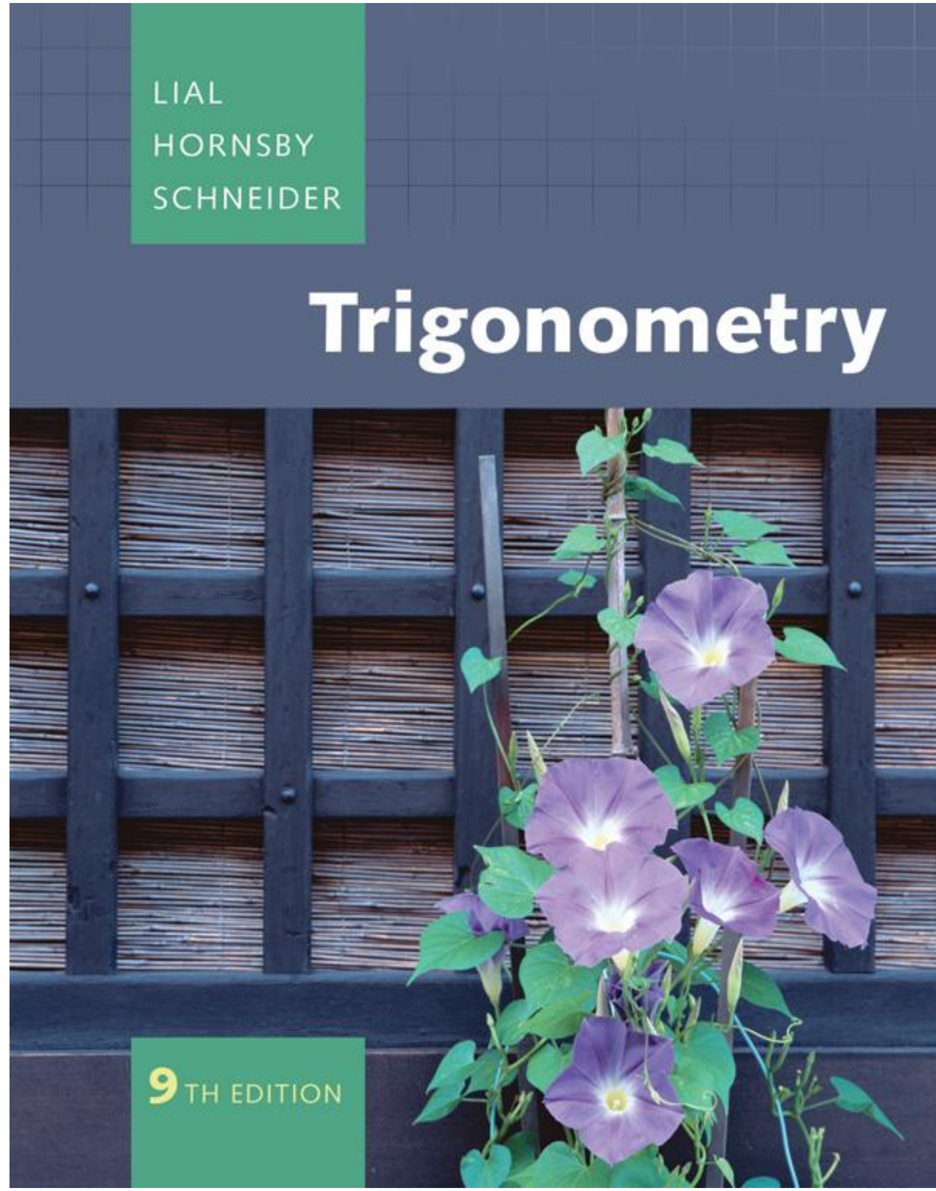
2

Acute Angles and Right Triangle



PEARSON Education

2 Acute Angles and Right Triangles

2.1 Trigonometric Functions of Acute Angles

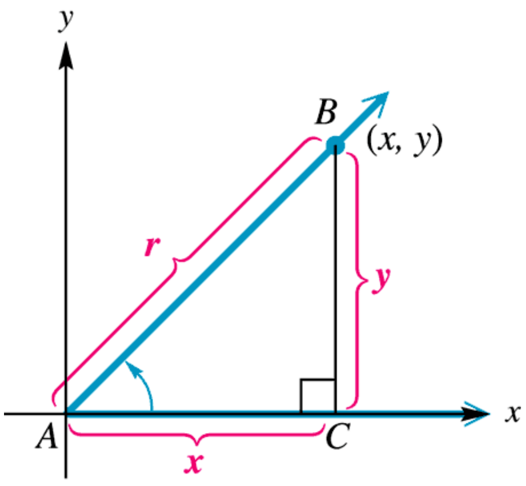
Right-Triangle-Based Definitions of the Trigonometric Functions
Cofunctions   Trigonometric Function Values of Special Angles

Right-Triangle-Based Definitions of Trigonometric Functions

For any acute angle *A* in standard position,

$$\sin A = \frac{y}{r} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\csc A = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite}}$$

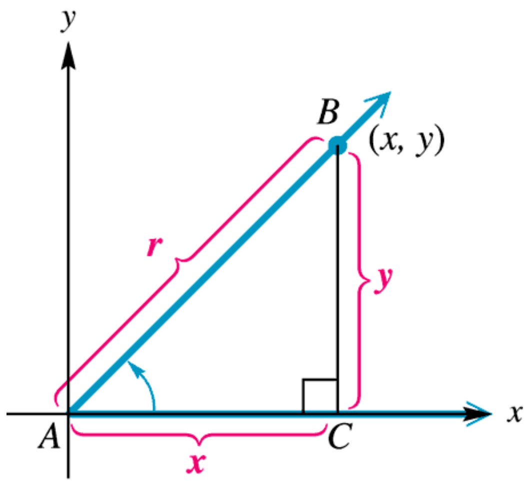


Right-Triangle-Based Definitions of Trigonometric Functions

For any acute angle *A* in standard position,

$$\cos A = \frac{x}{r} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\sec A = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent}}$$

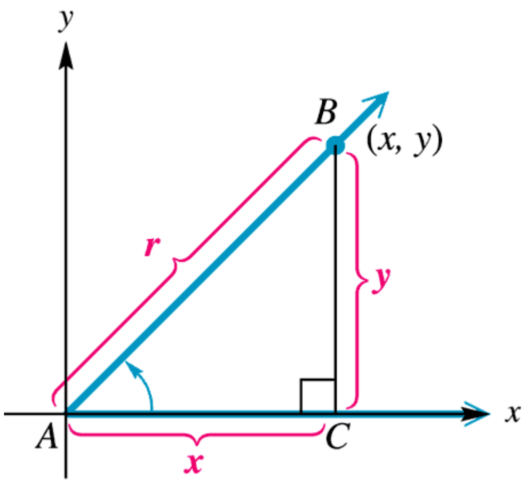


Right-Triangle-Based Definitions of Trigonometric Functions

For any acute angle *A* in standard position,

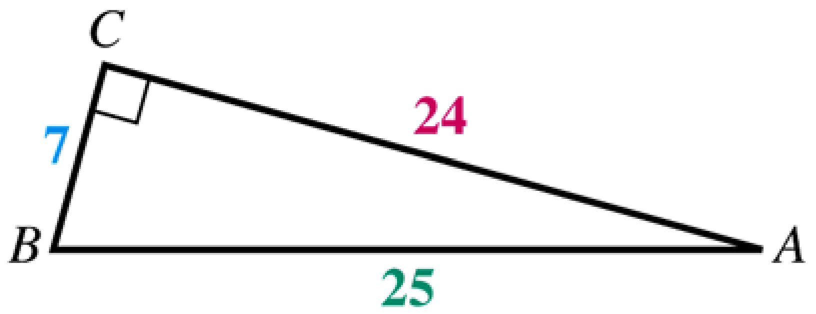
$$\tan A = \frac{y}{x} = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\cot A = \frac{x}{y} = \frac{\text{side adjacent}}{\text{side opposite}}$$



▶ Example 1 FINDING TRIGONOMETRIC FUNCTION VALUES OF AN ACUTE ANGLE

Find the sine, cosine, and tangent values for angles *A* and *B.*



$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{7}{25}$$

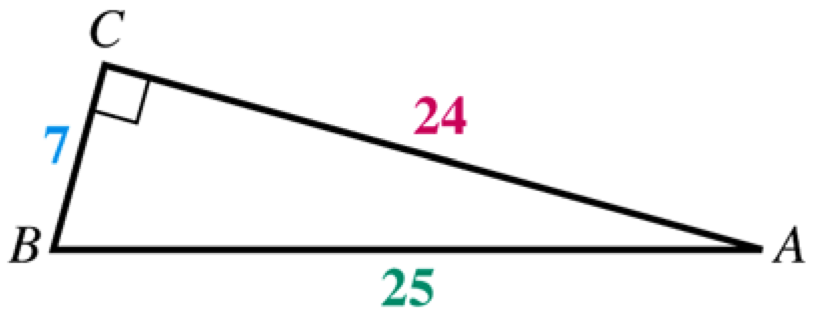
$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{7}{24}$$

▶ Example 1

FINDING TRIGONOMETRIC FUNCTION VALUES OF AN ACUTE ANGLE (cont.)

Find the sine, cosine, and tangent values for angles *A* and *B.*



$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{7}{25}$$

$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{24}{7}$$

Cofunction Identities

For any acute angle A in standard position,

$\sin A = \cos(90° - A)$ $\csc A = \sec(90° - A)$

$\tan A = \cot(90° - A)$ $\cos A = \sin(90° - A)$

$\sec A = \csc(90° - A)$ $\cot A = \tan(90° - A)$

Example 2 WRITING FUNCTIONS IN TERMS OF COFUNCTIONS

Write each function in terms of its cofunction.

(a) cos 52 = sin (90 – 52) = sin 38

(b) tan 71 = cot (90 – 71) = cot 19

(c) sec 24 = csc (90 – 24) = csc 66

Example 3 SOLVING EQUATIONS USING THE COFUNCTION IDENTITIES

Find one solution for the equation. Assume all angles involved are acute angles.

(a) $\cos(\theta + 4°) = \tan(4\theta + 13°)$

Since sine and cosine are cofunctions, the equation is true if the sum of the angles is 90º.

$$(\theta + 4°) + (3\theta + 2°) = 90°$$

$$4\theta + 6° = 90° \quad \text{Combine terms.}$$

$$4\theta = 84° \quad \text{Subtract 6 .}$$

$$\theta = 21° \quad \text{Divide by 4.}$$

Example 3 SOLVING EQUATIONS USING THE COFUNCTION IDENTITIES (continued)

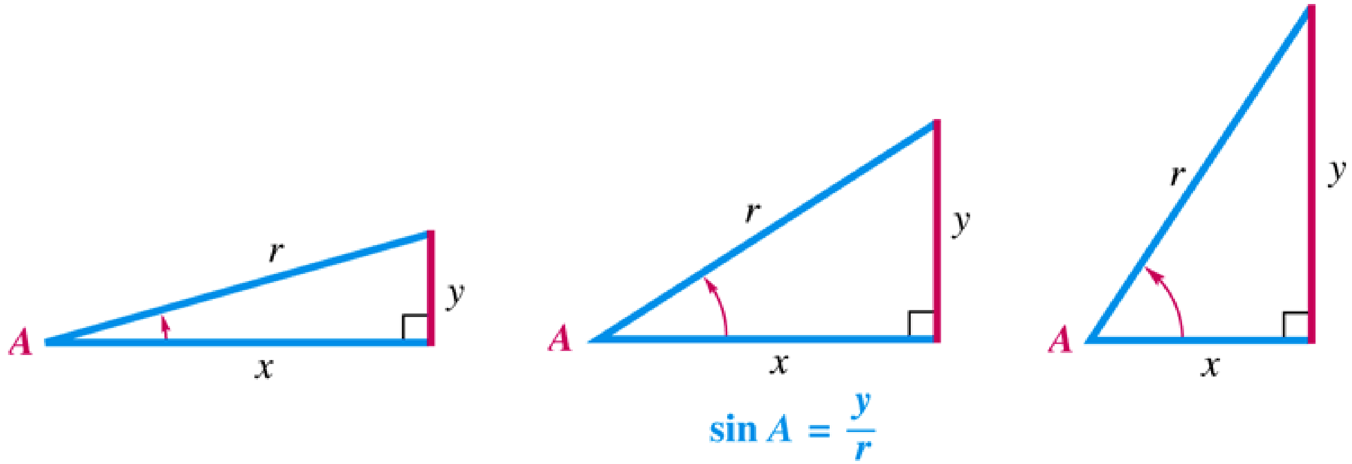
Find one solution for the equation. Assume all angles involved are acute angles.

(b) $\cos(\theta + 4^\circ) = \tan(4\theta + 13^\circ)$

Since tangent and cotangent are cofunctions, the equation is true if the sum of the angles is 90º.

$$(2\theta - 18^\circ) + (\theta + 18^\circ) = 90^\circ$$
$$3\theta = 90^\circ$$
$$\theta = 30^\circ$$

Increasing/Decreasing Functions



As A increases, y increases and x decreases.

Since r is fixed,

sin A increases

cos A decreases

tan A increases

csc A decreases

sec A increases

cot A decreases

Example 4 COMPARING FUNCTION VALUES OF ACUTE ANGLES

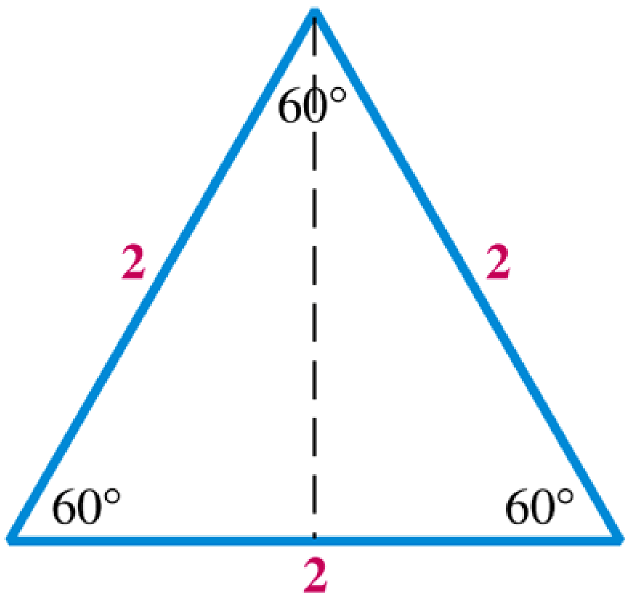
Determine whether each statement is *true* or *false*.

(a) sin 21 > sin 18 (b) cos 49 ≤ cos 56

(a) In the interval from 0° to 90°, as the angle increases, so does the sine of the angle, which makes sin 21 > sin 18 a true statement.

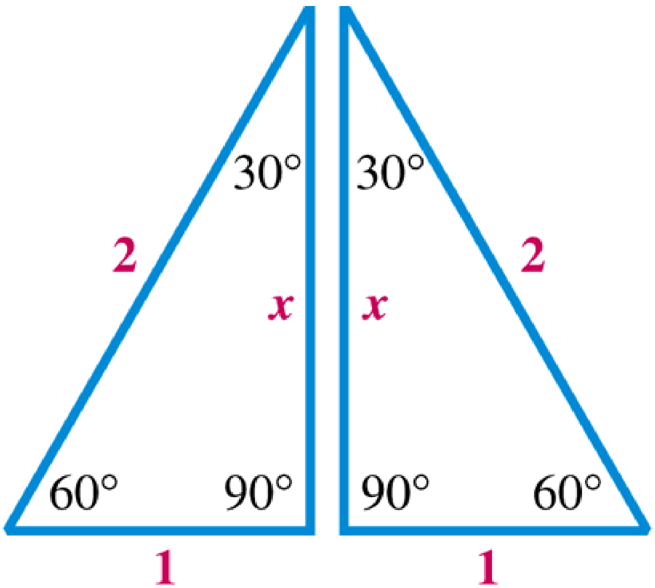
(b) In the interval from 0° to 90°, as the angle increases, the cosine of the angle decreases, which makes cos 49 ≤ cos 56 a false statement.

30 - 60 - 90 Triangles



Equilateral triangle

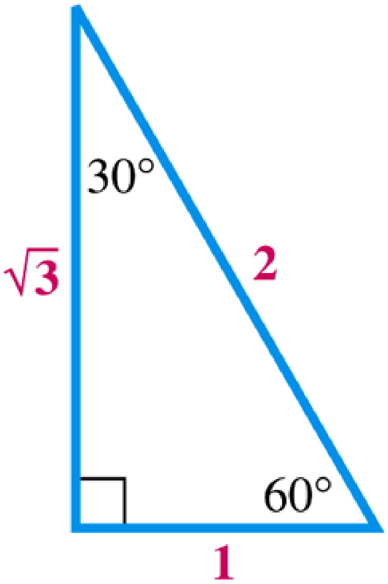
Bisect one angle of an equilateral to create two 30 -60 -90 triangles.



30°–60° right triangle

30 - 60 - 90 Triangles

Use the Pythagorean theorem to solve for *x*.



$$2^2 = 1^2 + x^2$$

$$4 = 1 + x^2$$

$$3 = x^2$$

$$\sqrt{3} = x$$

▶ Example 5

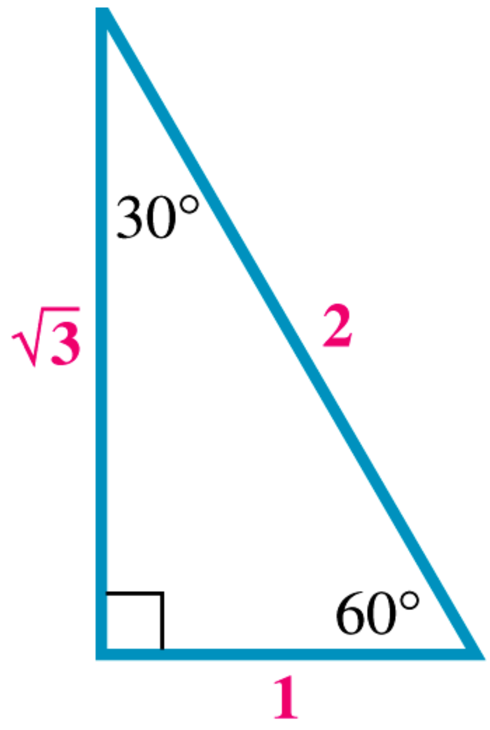
FINDING TRIGONOMETRIC FUNCTION VALUES FOR 60

Find the six trigonometric function values for a 60 angle.

$$\sin 60^\circ = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$



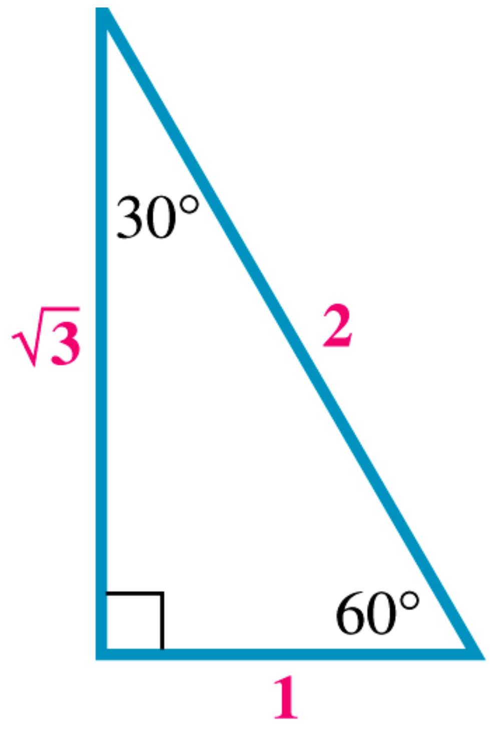
Example 5 FINDING TRIGONOMETRIC FUNCTION VALUES FOR 60 (continued)

Find the six trigonometric function values for a 60 angle.

$$\cot 60^\circ = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec 60^\circ = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{2}{1} = 2$$

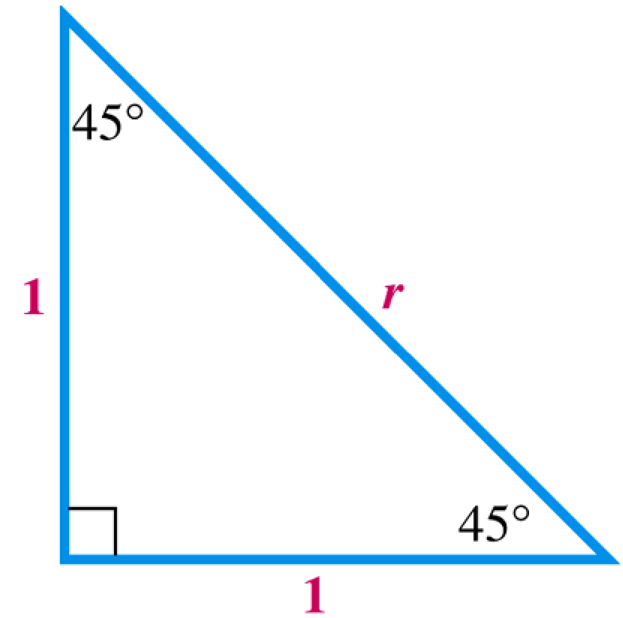
$$\csc 60^\circ = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$



45 - 45 Right Triangles

Use the Pythagorean theorem to solve for *r*.

$$1^2 + 1^2 = r^2$$
$$2 = r^2$$
$$\sqrt{2} = r$$



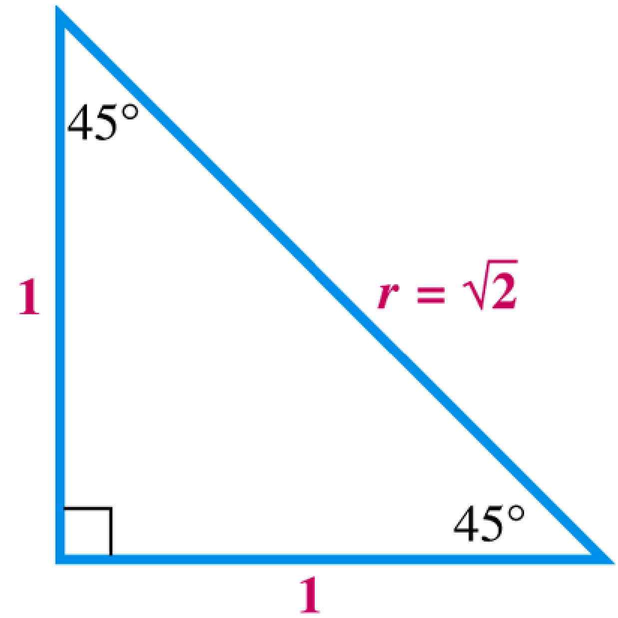
45°– 45° right triangle

45 - 45 Right Triangles

$$\sin 45° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{1}{1} = 1$$



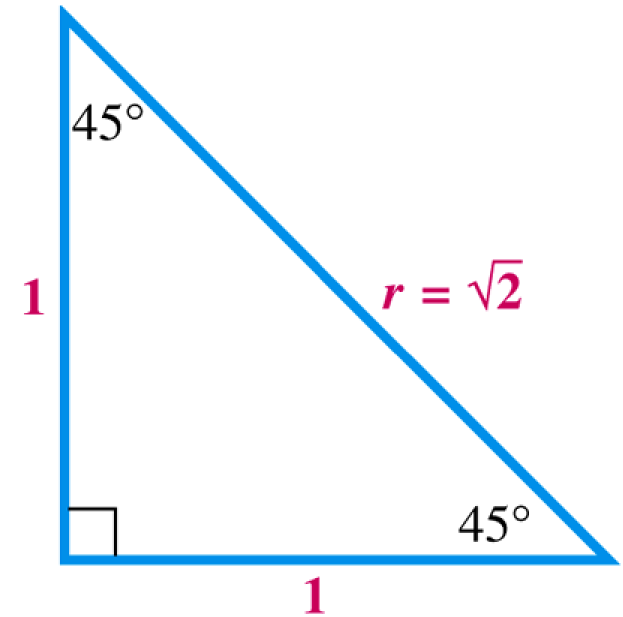
45°–45° right triangle

45 - 45 Right Triangles

$$\cot 45° = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{1}{1} = 1$$

$$\sec 45° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\csc 45° = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$



45°– 45° right triangle

Function Values of Special Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$

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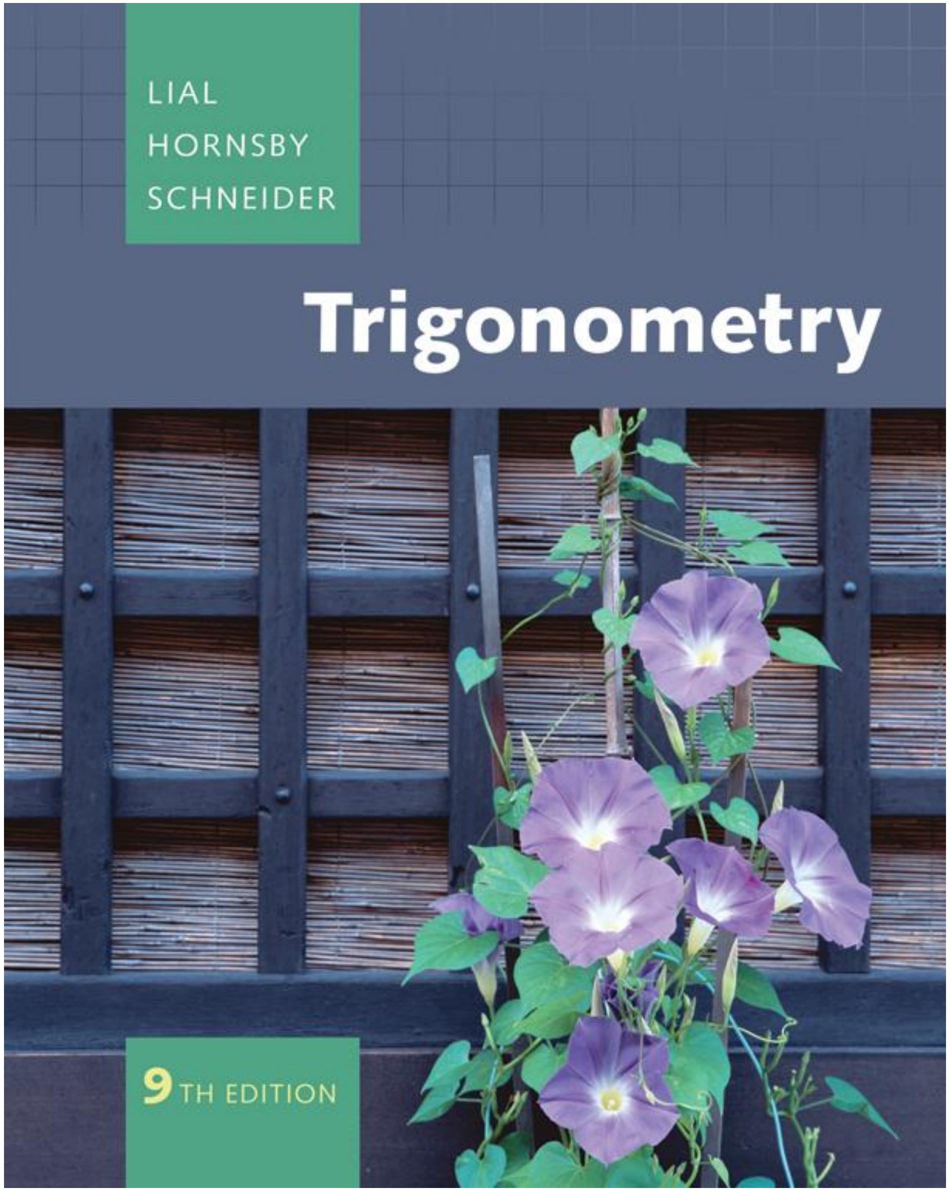
Acute Angles and Right Triangle



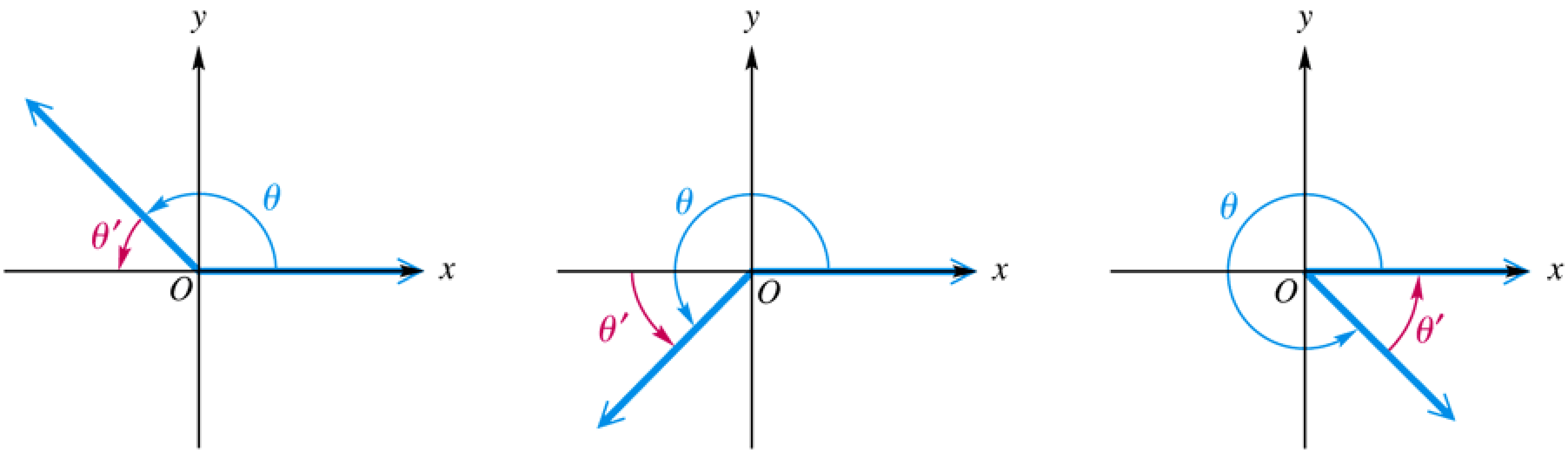
2 Acute Angles and Right Triangles

2.2 Trigonometric Functions of Non-Acute Angles

Reference Angles   Special Angles as Reference Angles
Finding Angle Measures with Special Angles

Reference Angles

A reference angle for an angle θ is the positive acute angle made by the terminal side of angle θ and the x-axis.



θ in quadrant II $\quad$ θ in quadrant III $\quad$ θ in quadrant IV

Caution

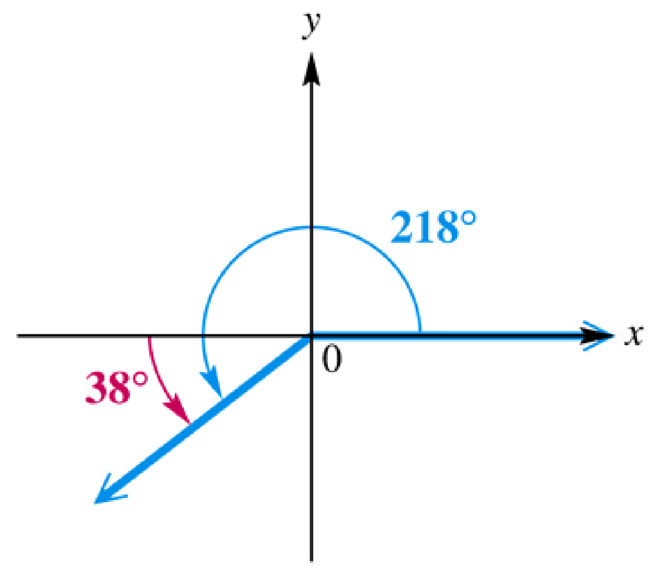
A common error is to find the reference angle by using the terminal side of θ and the y-axis.

The reference angle is always found with reference to the x-axis.

Example 1(a) FINDING REFERENCE ANGLES

Find the reference angle for an angle of 218 .

The positive acute angle made by the terminal side of the angle and the *x*-axis is 218 – 180 = 38 .



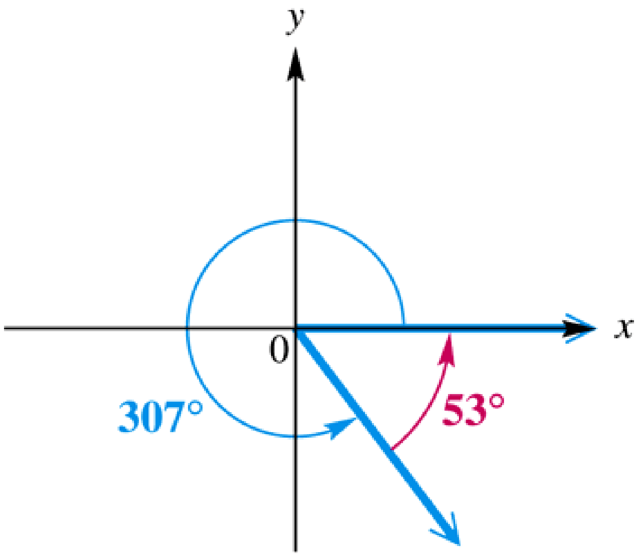
For θ = 218 , the reference angle θ' = 38 .

Example 1(b) FINDING REFERENCE ANGLES

Find the reference angle for an angle of 1387 .

First find a coterminal angle between 0 and 360 .

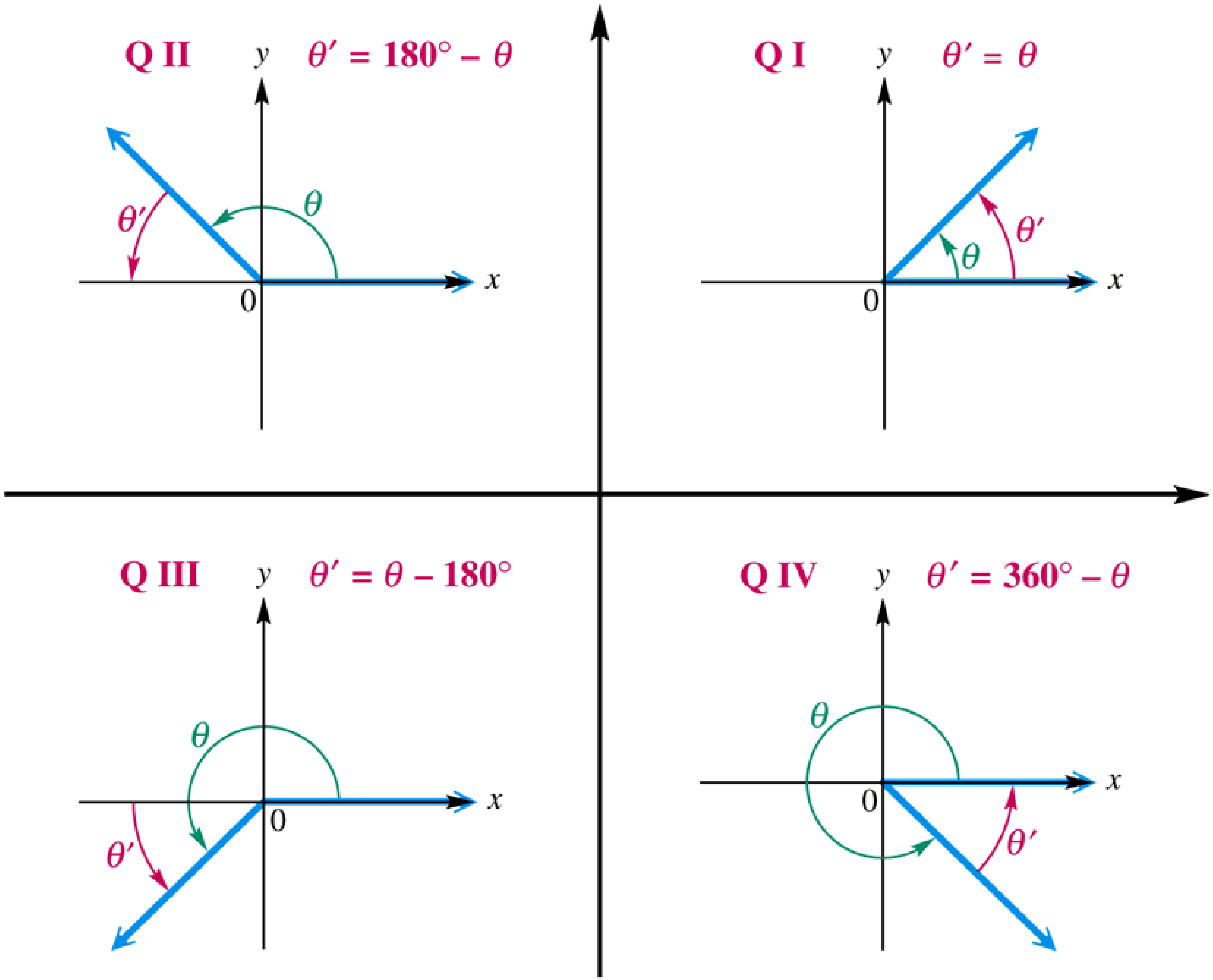
Divide 1387 by 360 to get a quotient of about 3.9. Begin by subtracting 360 three times.
1387 – 3(360) = 307 .



360° – 307° = 53°

The reference angle for 307 (and thus for 1387) is 360 – 307 = 53 .

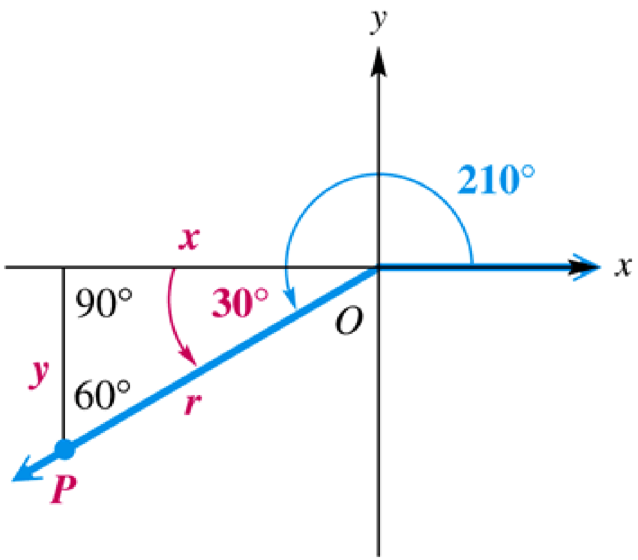
Reference Angle θ' for θ, where $0° < \theta < 360°$*



Example 2 FINDING TRIOGNOMETRIC FUNCTION VALUES OF A QUADRANT III ANGLE

Find the values of the six trigonometric functions for 210 .

The reference angle for a 210° angle is 210 – 180 = 30 .



Choose point *P* on the terminal side of the angle so the distance from the origin to *P* is 2.

$$r = 2, \ x = -\sqrt{3}, \ y = 1$$

Example 2

FINDING TRIOGNOMETRIC FUNCTION VALUES OF A QUADRANT III ANGLE (continued)

$$\sin 210° = \frac{-1}{2} = -\frac{1}{2}$$

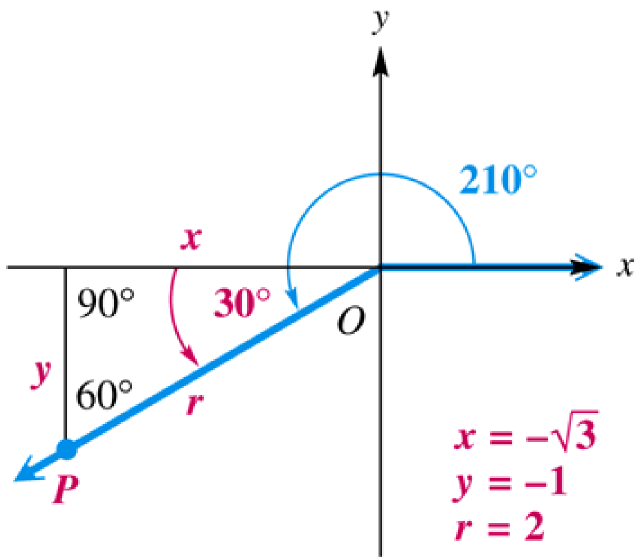
$$\cos 210° = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$\tan 210° = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot 210° = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

$$\sec 210° = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\csc 210° = \frac{2}{-1} = -2$$



Finding Trigonometric Function Values For Any Nonquadrantal Angle θ

Step 1 If $\theta > 360$, or if $\theta < 0°$, find a coterminal angle by adding or subtracting 360 as many times as needed to get an angle greater than 0 but less than 360 .

Step 2 Find the reference angle θ'.

Step 3 Find the trigonometric function values for reference angle θ'.

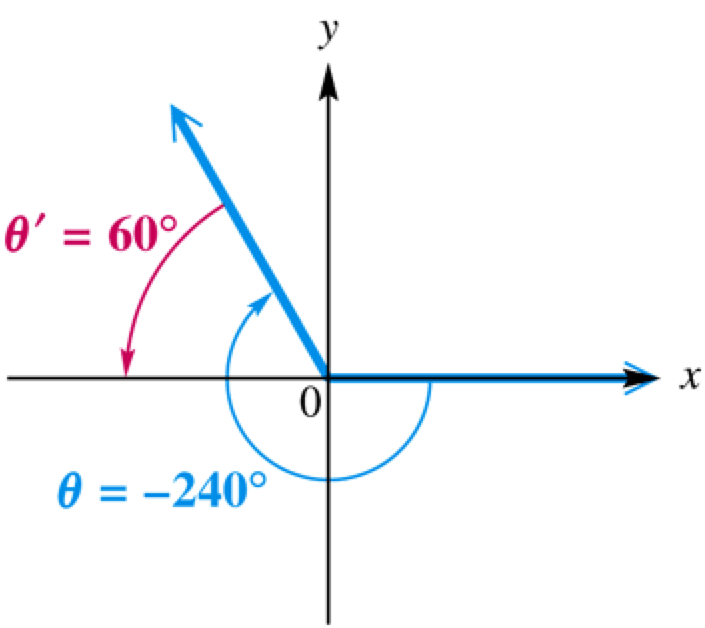
Finding Trigonometric Function Values For Any Nonquadrantal Angle θ (continued)

Step 4 Determine the correct signs for the values found in *Step 3*. This gives the values of the trigonometric functions for angle θ.

Example 3(a) FINDING TRIGONOMETRIC FUNCTION VALUES USING REFERENCE ANGLES

Find the exact value of cos (–240).

Since an angle of –240 is coterminal with an angle of –240 + 360 = 120 , the reference angle is 180 – 120 = 60 .

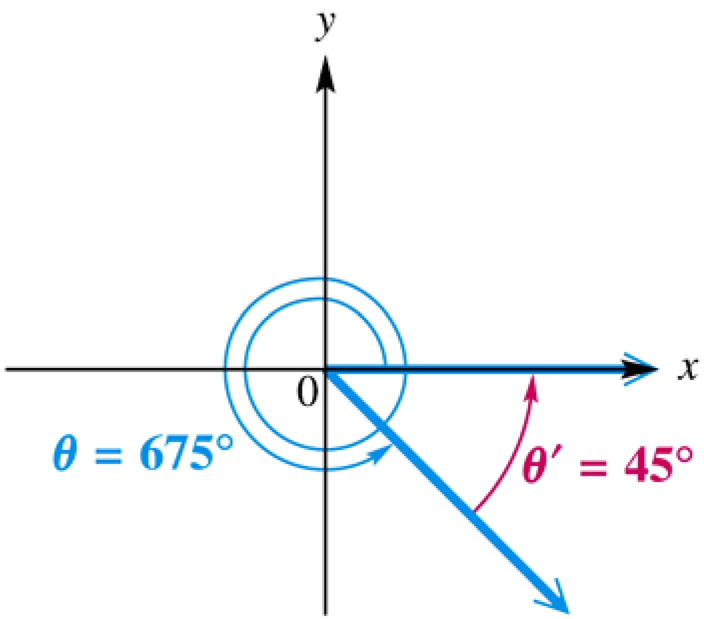


$$
\begin{aligned}
\cos(-240^\circ) &= \cos 120^\circ \\
&= -\cos 60^\circ \\
&= -\frac{1}{2}
\end{aligned}
$$

Example 3(b) FINDING TRIGONOMETRIC FUNCTION VALUES USING REFERENCE ANGLES

Find the exact value of tan 675 .

Subtract 360 to find a coterminal angle between 0 and 360 : 657 – 360 = 315 .



$$\tan 675^\circ = \tan 315^\circ$$
$$= -\tan 45^\circ$$
$$= -1$$

Example 4

EVALUATING AN EXPRESSION WITH FUNCTION VALUES OF SPECIAL ANGLES

Evaluate $\cos 120^\circ + 2\sin^2 60^\circ - \tan^2 30^\circ$.

$$\cos 120^\circ = -\frac{1}{2} \qquad \sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$\begin{aligned}\cos 120^\circ + 2\sin^2 60^\circ - \tan^2 30^\circ &= -\frac{1}{2} + 2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{3}\right)^2 \\ &= -\frac{1}{2} + 2\left(\frac{3}{4}\right) - \frac{3}{9} \\ &= \frac{2}{3}\end{aligned}$$

Example 5(a) USING COTERMINAL ANGLES TO FIND FUNCTION VALUES

Evaluate cos 780 by first expressing the function in terms of an angle between 0° and 360°.

$$\cos 780^\circ = \cos(780^\circ - 2 \cdot 360^\circ)$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

Example 5(b) USING COTERMINAL ANGLES TO FIND FUNCTION VALUES

Evaluate tan (–405) by first expressing the function in terms of an angle between 0° and 360°.

$$\tan(-405°) = \tan(-405° + 2 \cdot 360°)$$

$$= \tan 315°$$

315 lies in quadrant IV. Its reference angle is 45

$$= -\tan 45°$$

$$= -1$$

▶ Example 6

FINDING ANGLE MEASURES GIVEN AN INTERVAL AND A FUNCTION VALUE

Find all values of *θ*, if *θ* is in the interval [0 , 360) and $\cos\theta = -\frac{\sqrt{2}}{2}$.

Since cos *θ* is negative, *θ* must lie in quadrants II or III.

The absolute value of cos *θ* is $\frac{\sqrt{2}}{2}$, so the reference angle is 45 .

The angle in quadrant II is 180 – 45 = 135 .

The angle in quadrant III is 180 + 45 = 225 .

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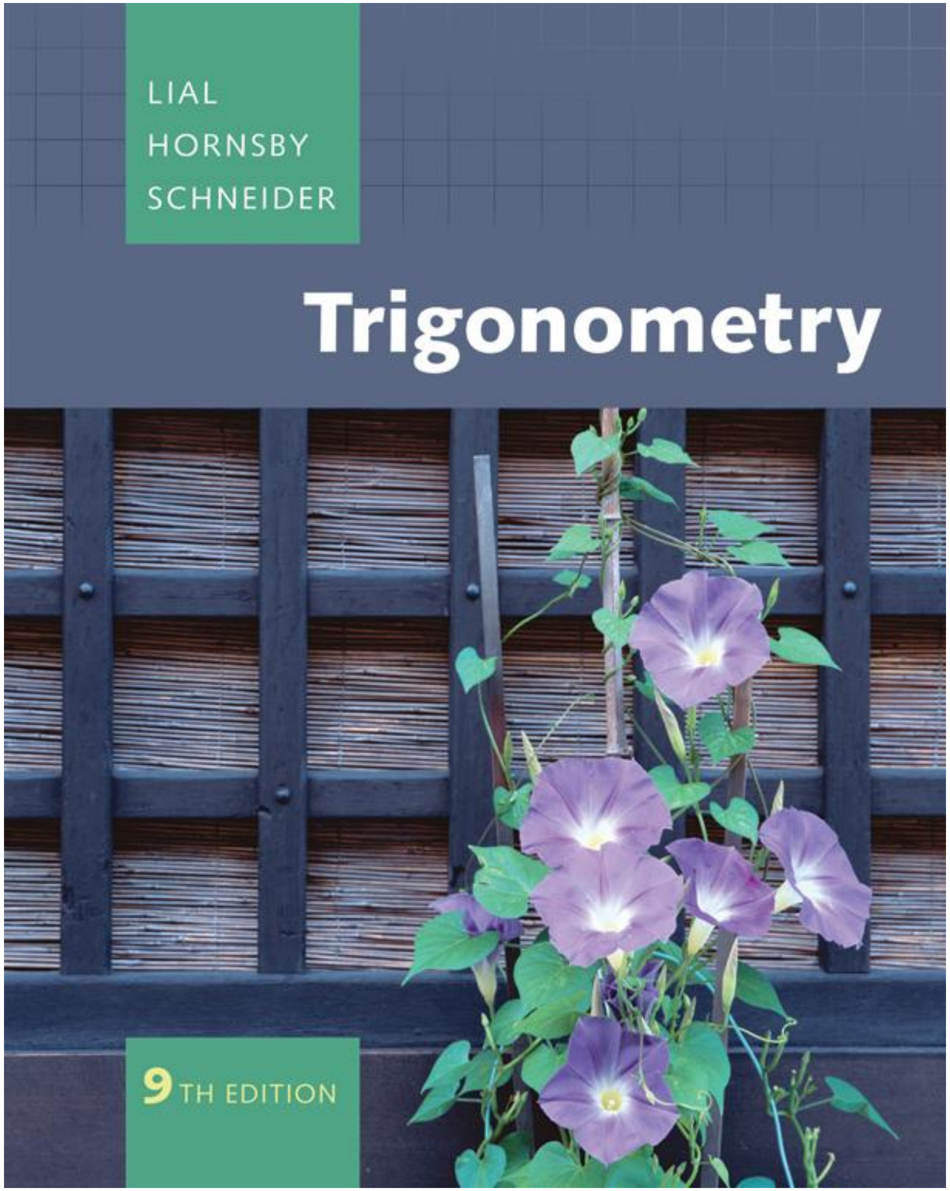
Acute Angles and Right Triangle



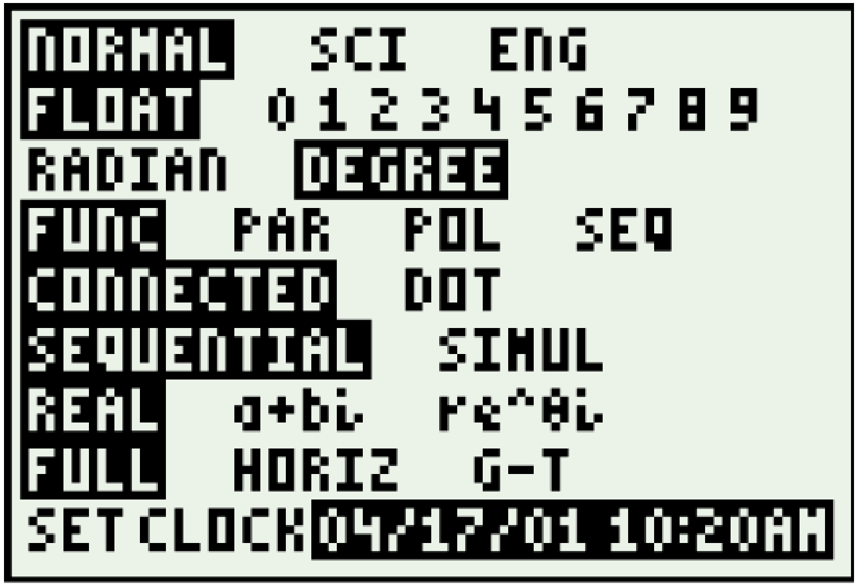
2 Acute Angles and Right Triangles

2.3 Finding Trigonometric Function Values Using a Calculator

Finding Function Values Using a Calculator ▪ Finding Angle Measures Using a Calculator

Caution

When evaluating trigonometric functions of angles given in degrees, remember that the calculator must be set in degree mode.



Example 1

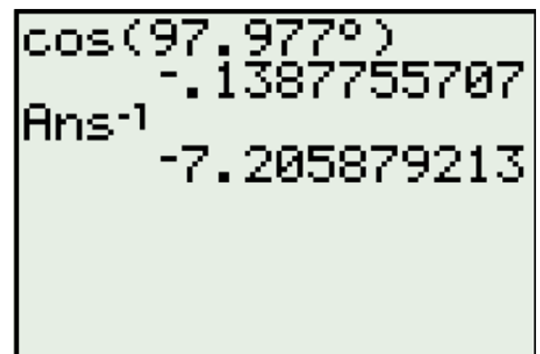
FINDING FUNCTION VALUES WITH A CALCULATOR

Approximate the value of each expression.

(a) sin 49 12′ ≈ .75699506

(b) sec 97.977

Calculators do not have a secant key, so first find cos 97.977° and then take the reciprocal.

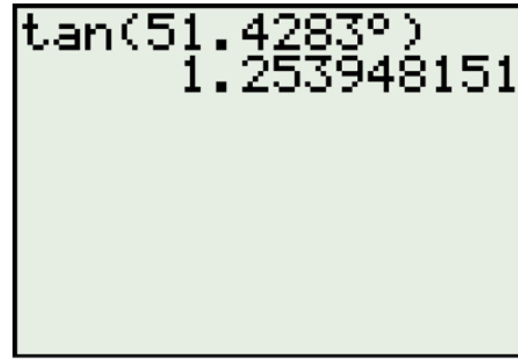


sec 97.977 ≈ –.75699506

Example 1 FINDING FUNCTION VALUES WITH A CALCULATOR (continued)

Approximate the value of each expression.

(c) $\dfrac{1}{\cot 51.4283°}$

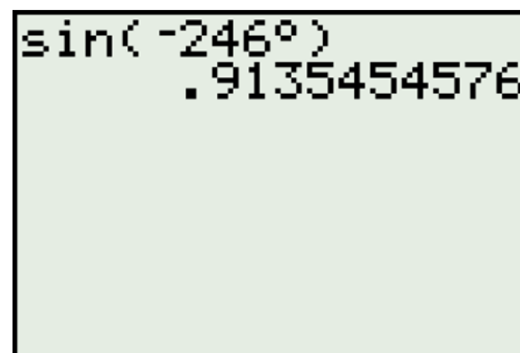


Use the reciprocal identity

$$\tan\theta = \frac{1}{\cot\theta}.$$

$$\frac{1}{\cot 51.4283°} = \tan 51.4283° \approx 1.25394815$$

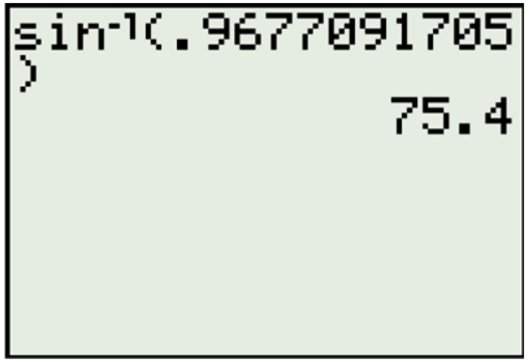
(d) sin (–246) ≈ –.91354546



Example 2 USING INVERSE TRIGONOMETRIC FUNCTIONS TO FIND ANGLES

Use a calculator to find an angle θ in the interval [0 , 90] that satisfies each condition.

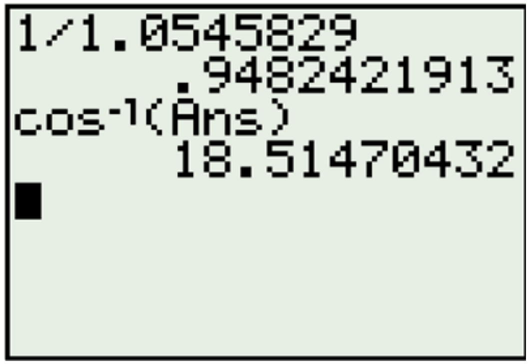
(a) $\sin\theta \approx .9677091705$



Use degree mode and the inverse sine function.

$\theta \approx \sin^{-1}.9677091705 \approx 75.4^\circ$

(b) $\sec\theta \approx 1.0545829$



Use the identity $\cos\theta = \dfrac{1}{\sec\theta}$.

$\theta \approx 18.514704^\circ$

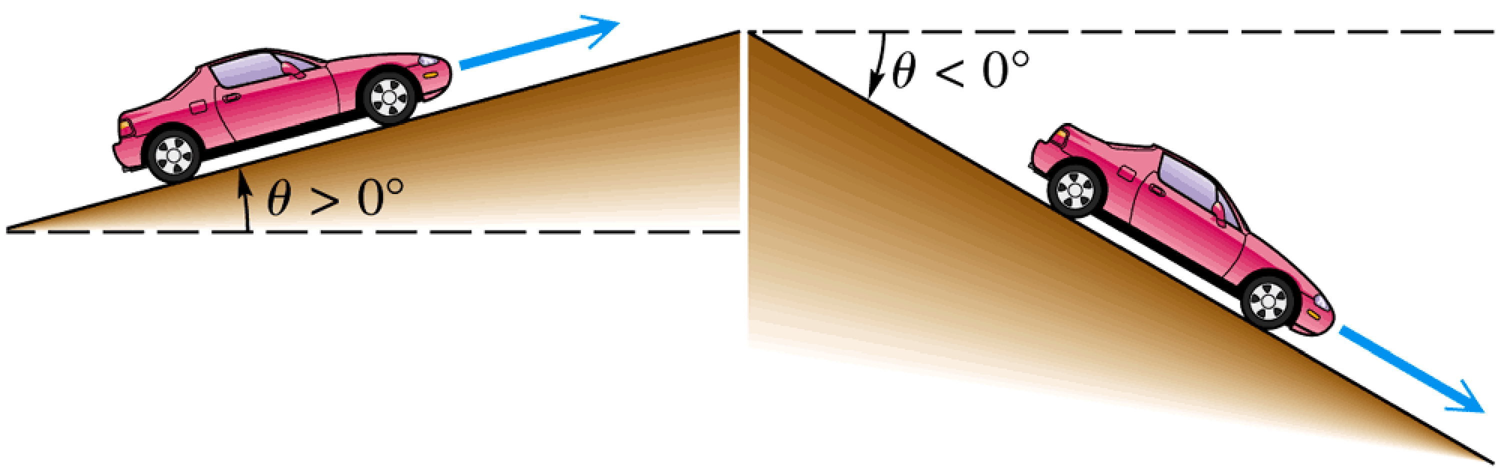
Caution

Note that the reciprocal is used **before** ***the inverse trigonometric function key when finding the angle, but*** **after** ***the trigonometric function key when finding the trigonometric function value.***

Example 3 FINDING GRADE RESISTANCE

The force F in pounds when an automobile travels uphill or downhill on a highway is called **grade resistance** and is modeled by the equation $F = W \sin \theta$, where θ is the grade and W is the weight of the automobile.

If the automobile is moving uphill, then $\theta > 0$; if it is moving downhill, then $\theta < 0$.



Example 3 FINDING GRADE RESISTANCE (cont.)

(a) Calculate F to the nearest 10 pounds for a 2500-lb car traveling an uphill grade with $\theta = 2.5$.

$$F = W\sin\theta = 2500\sin 2.5^\circ \approx 110 \text{ lb}$$

(b) Calculate F to the nearest 10 pounds for a 5000-lb truck traveling a downhill grade with $\theta = -6.1$.

$$F = W\sin\theta = 5000\sin(-6.1^\circ) \approx -530 \text{ lb}$$

Example 3 FINDING GRADE RESISTANCE (cont.)

(c) Calculate F for $\theta = 0$ and $\theta = 90$.

$$F = W \sin\theta = W \sin 0^\circ = W(0) = 0 \text{ lb}$$

If $\theta = 0$, then the road is level and gravity does not cause the vehicle to roll.

$$F = W \sin\theta = W \sin 90^\circ = W(1) = W \text{ lb}$$

If $\theta = 90$, then the road is vertical and the full weight of the vehicle would be pulled downward by gravity.

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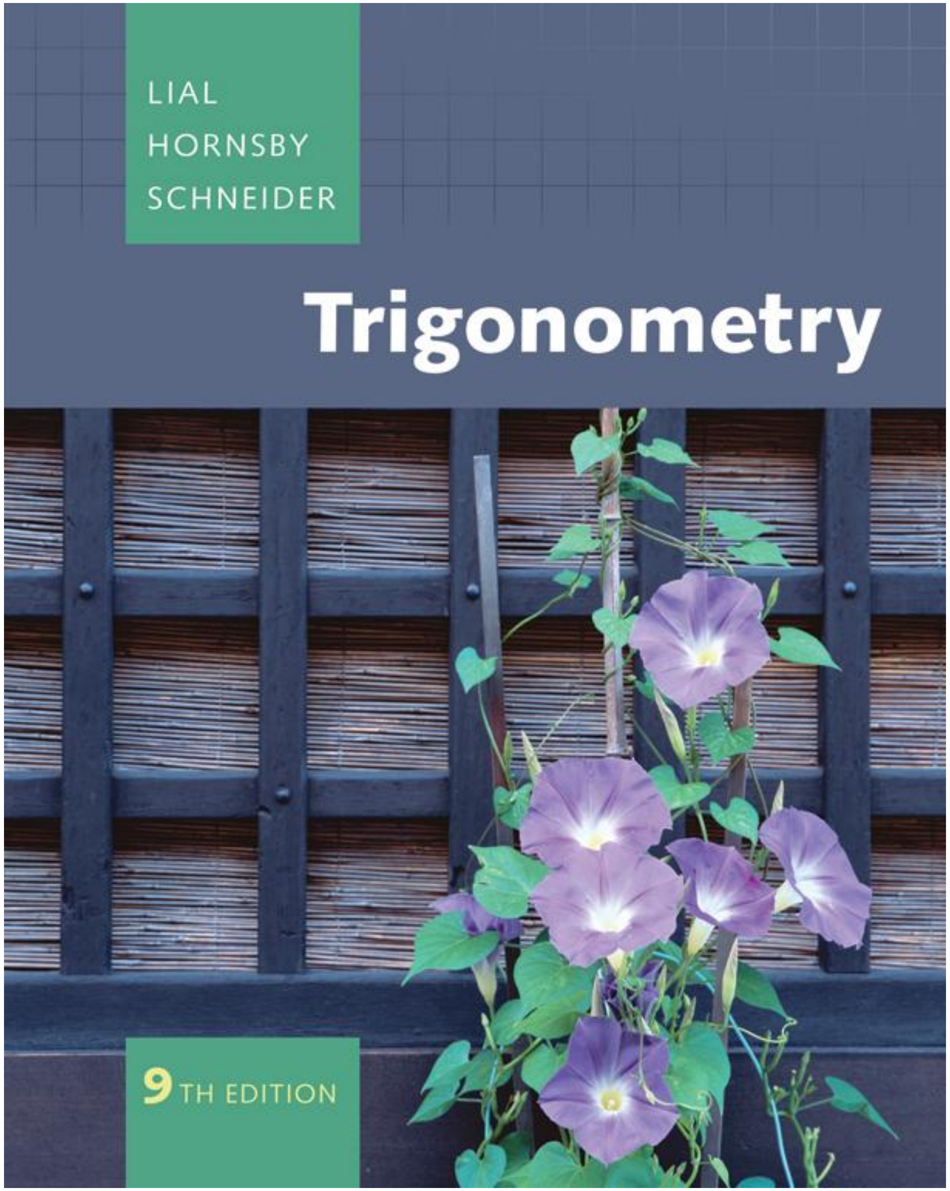
Acute Angles and Right Triangle



2 Acute Angles and Right Triangles

2.4 Solving Right Triangles

Significant Digits   Solving Triangles   Angles of Elevation or Depression

Significant Digits

A significant digit is a digit obtained by actual measurement.

The significant digits in the following numbers are identified in color.

408   21.5   18.00   6.700   .0025   .09810   7300

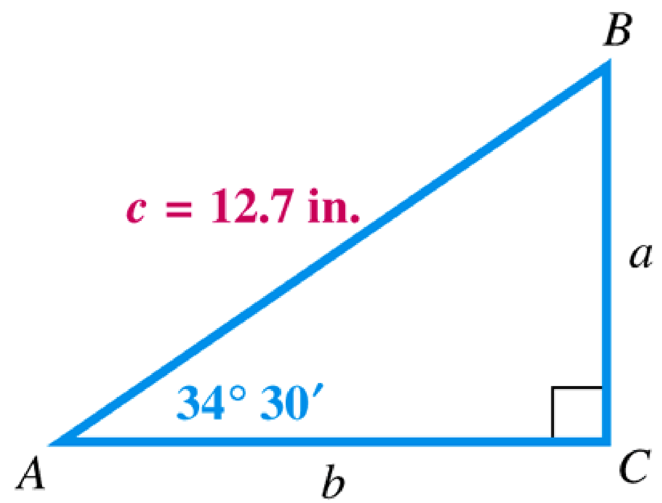
Your answer is no more accurate then the least accurate number in your calculation.

To determine the number of significant digits for answers in applications of angle measure, use the table below.

Angle Measure to Nearest	Examples	Answer to Number of Significant Digits
Degree	$62°$, $36°$	2
Ten minutes, or nearest tenth of a degree	$52° 30'$, $60.4°$	3
Minute, or nearest hundredth of a degree	$81° 48'$, $71.25°$	4
Ten seconds, or nearest thousandth of a degree	$10° 52' 20''$, $21.264°$	5

Example 1 SOLVING A RIGHT TRIANGLE GIVEN AN ANGLE AND A SIDE

Solve right triangle *ABC*, if *A* = 34 30′ and *c* = 12.7 in.



$$\sin A = \frac{a}{c}$$

$$\sin 34^\circ 30' = \frac{a}{12.7}$$

$$a = 12.7 \sin 34^\circ 30' \approx 7.19 \text{ in.}$$

$$\cos A = \frac{b}{c}$$

$$\cos 34^\circ 30' = \frac{b}{12.7}$$

$$b = 12.7 \cos 34^\circ 30' \approx 10.5 \text{ in.}$$

$$B = 90^\circ - A$$

$$B = 90^\circ - 34^\circ 30'$$

$$B = 55^\circ 30'$$

▶ Example 1 SOLVING A RIGHT TRIANGLE GIVEN AN ANGLE AND A SIDE (continued)

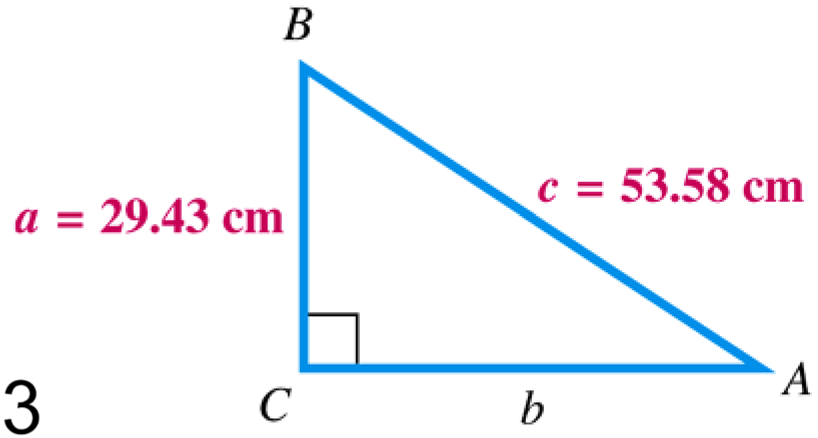
We could have found the measure of angle *B* first, and then used the trigonometric functions of *B* to find the unknown sides.

The process of solving a right triangle can usually be done in several ways, each producing the correct answer.

To maintain accuracy, always use given information as much as possible, and avoid rounding off in intermediate steps.

Example 2 SOLVING A RIGHT TRIANGLE GIVEN TWO SIDES

Solve right triangle *ABC*, if $a = 29.43$ cm and $c = 53.58$ cm.



$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{29.43}{53.58}$$

$$A = \sin^{-1}\left(\frac{29.43}{53.58}\right) \approx 33.32°$$

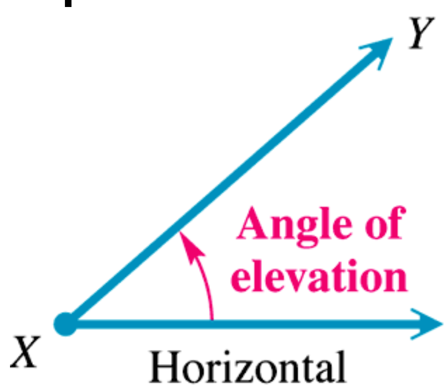
$$B \approx 90° - 33.32° = 56.68°$$

$$b^2 = c^2 - a^2 \Rightarrow b^2 = 53.58^2 - 29.43^2$$

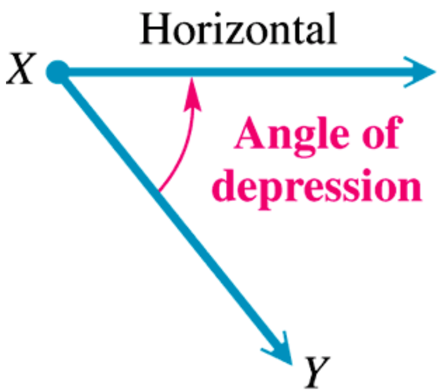
$$b = \sqrt{2004.6915} \approx 44.77 \text{ cm}$$

Angles of Elevation or Depression

The ***angle of elevation*** from point *X* to point *Y* (above *X*) is the acute angle formed by ray *XY* and a horizontal ray with endpoint *X*.



The ***angle of depression*** from point *X* to point *Y* (below *X*) is the acute angle formed by ray *XY* and a horizontal ray with endpoint *X*.



Caution

Be careful when interpreting the angle of depression.

Both the angle of elevation and the angle of depression are measured between the line of sight and a horizontal line.

Solving an Applied Trigonometry Problem

Step 1 Draw a sketch, and label it with the given information. Label the quantity to be found with a variable.

Step 2 Use the sketch to write an equation relating the given quantities to the variable.

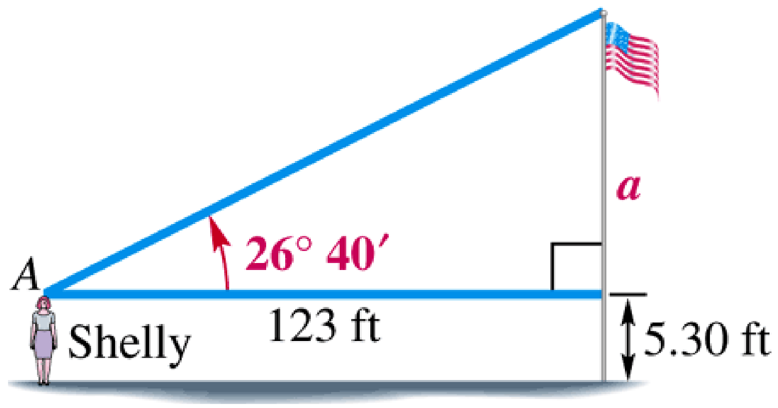
Step 3 Solve the equation, and check that your answer makes sense.

▶ Example 3 FINDING A LENGTH WHEN THE ANGLE OF ELEVATION IS KNOWN

When Shelly stands 123 ft from the base of a flagpole, the angle of elevation to the top of the flagpole is 26 40′. If her eyes are 5.30 ft above the ground, find the height of the flagpole.

$$\tan 26°40' = \frac{a}{123}$$

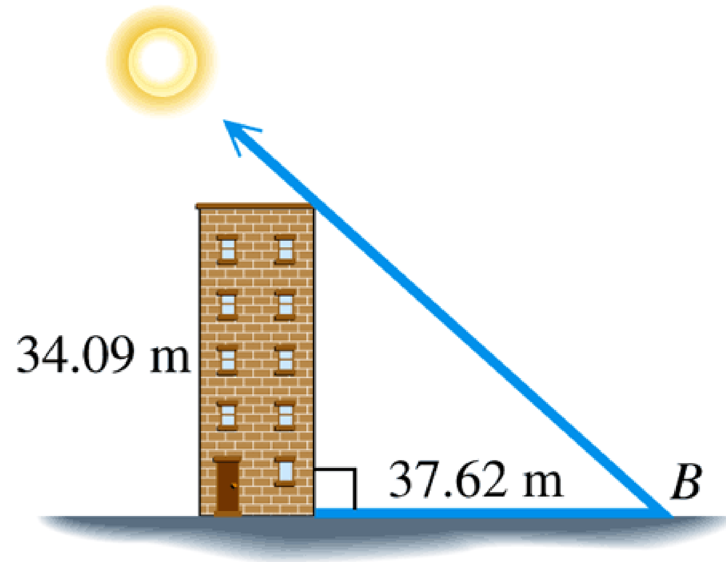
$$a = 123 \tan 26°40' \approx 61.8 \text{ ft}$$



Since Shelly’s eyes are 5.30 ft above the ground, the height of the flagpole is 61.8 + 5.30 = 67.1 ft

Example 4 FINDING THE ANGLE OF ELEVATION WHEN LENGTHS ARE KNOWN

The length of the shadow of a building 34.09 m tall is 37.62 m. Find the angle of elevation of the sun.

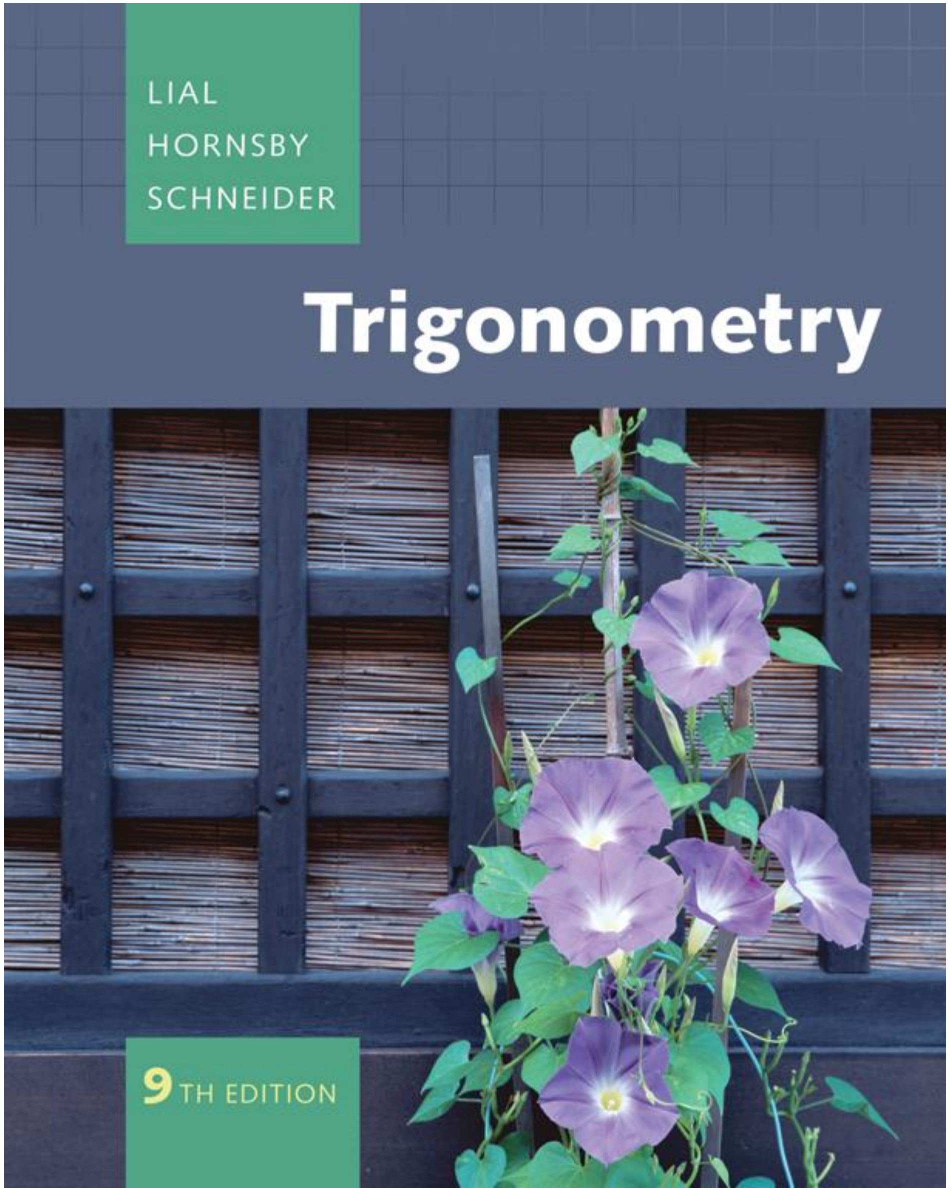


$$\tan B = \frac{34.09}{37.62} \Rightarrow B = \sin^{-1}\left(\frac{34.09}{37.62}\right) \approx 42.18°$$

The angle of elevation is about 42.18 .

2

Acute Angles and Right Triangle



2 Acute Angles and Right Triangles

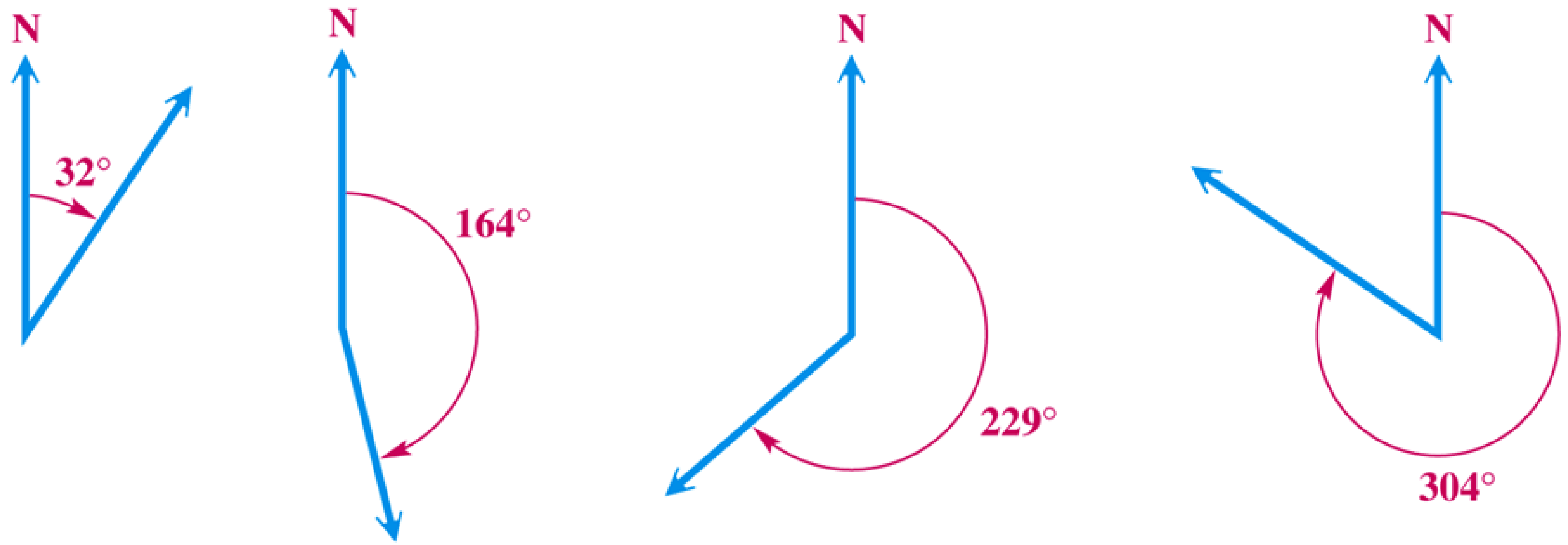
2.5 Further Applications of Right Triangles

Bearing   Further Applications

Bearing

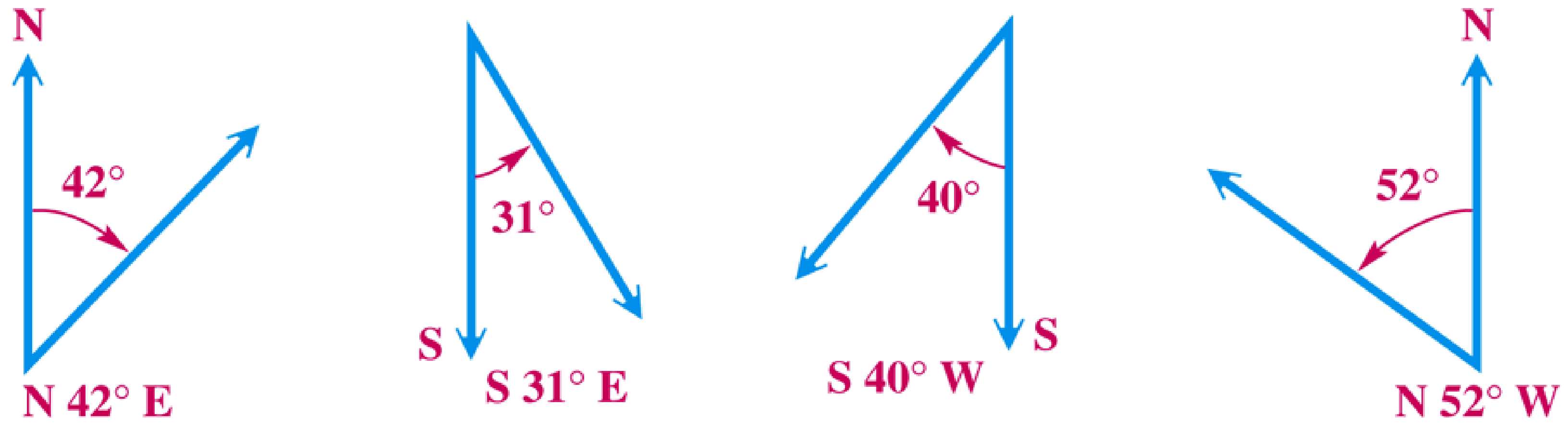
There are two methods for expressing bearing

When a single angle is given, such as 164 , it is understood that the bearing is measured in a clockwise direction from due north.



Bearing

The second method for expressing bearing starts with a norht-south line and uses an acute angle to show the direction, either east or west, from this line.

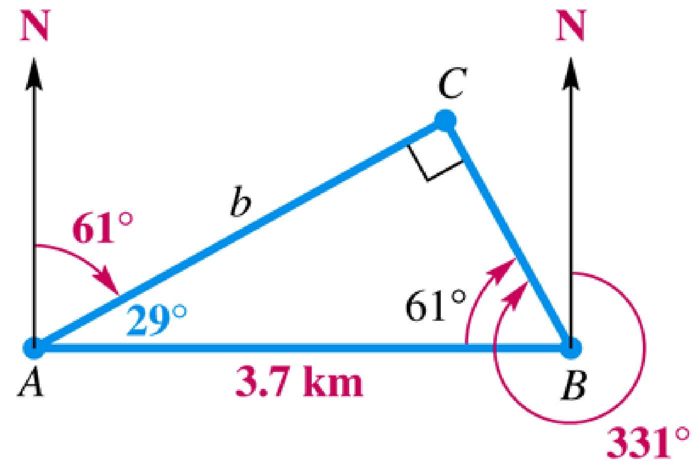


Example 1 SOLVING A PROBLEM INVOLVING BEARING (FIRST METHOD)

Radar stations *A* and *B* are on an east-west line, 3.7 km apart. Station *A* detects a plane at *C*, on a bearing on 61 . Station *B* simultaneously detects the same plane, on a bearing of 331 . Find the distance from *A* to *C*.

$$\cos 29° = \frac{b}{3.7}$$

$$b = 3.7 \cos 29° \approx 3.2 \text{ km}$$



Caution

A correctly labeled sketch is crucial when solving bearing applications.

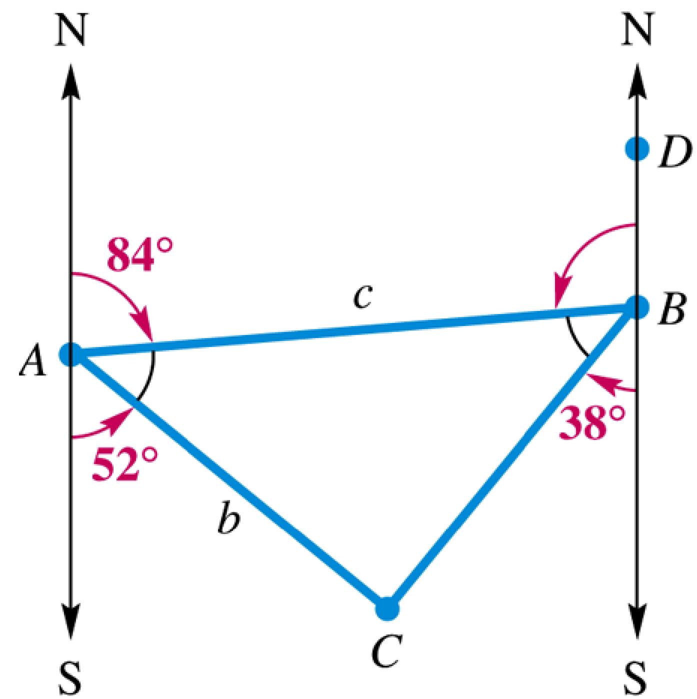
Some of the necessary information is often not directly stated in the problem and can be determined only from the sketch.

Example 2 SOLVING PROBLEM INVOLVING BEARING (SECOND METHOD)

The bearing from *A* to *C* is S 52 E. The bearing from *A* to *B* is N 84 E. The bearing from *B* to *C* is S 38 W. A plane flying at 250 mph takes 2.4 hours to go from *A* to *B*. Find the distance from *A* to *C*.

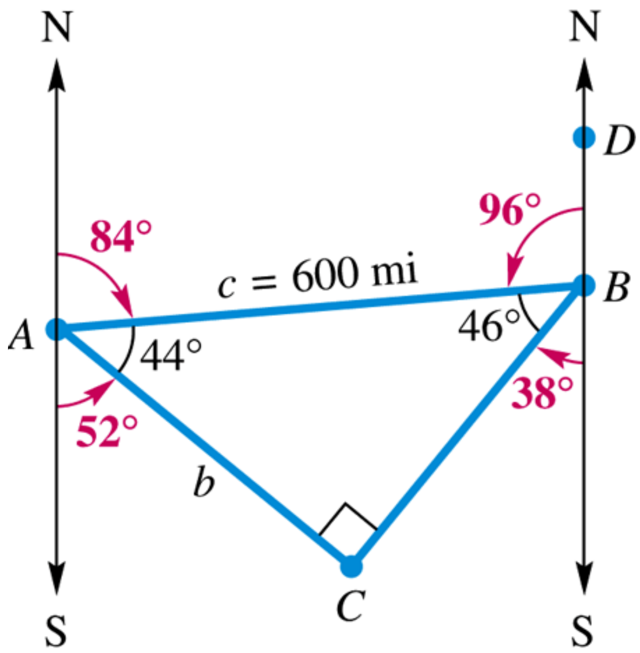
To draw the sketch, first draw the two bearings from point *A*.

Choose a point *B* on the bearing N 84 E from *A*, and draw the bearing to *C*, which is located at the intersection of the bearing lines from *A* and *B*.



▶ Example 2

SOLVING PROBLEM INVOLVING BEARING (SECOND METHOD) (cont.)



$$m\angle ABD = 180° - 84° = 96°$$

$$m\angle ABC = 180° - (96° + 38°)$$
$$= 46°$$

$$m\angle C = 180° - (44° + 46°)$$
$$= 90°$$

$$c = \text{rate} \times \text{time} = 250(2.4) = 600 \text{ mi.}$$

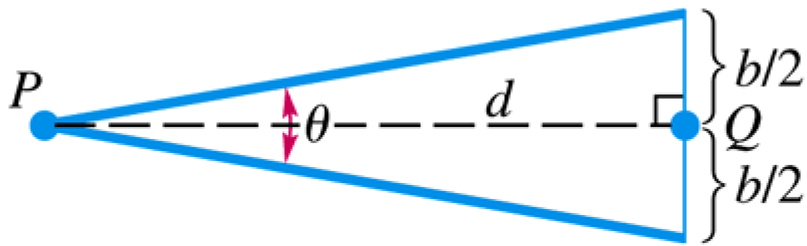
$$\sin 46° = \frac{b}{c} = \frac{b}{600}$$

$$b = 600 \sin 46° \approx 430 \text{ mi}$$

The distance from *A* to *B* is about 430 miles.

▶ Example 3 USING TRIGONOMETRY TO MEASURE A DISTANCE

A method that surveyors use to determine a small distance *d* between two points *P* and *Q* is called the **subtense bar method**. The subtense bar with length *b* is centered at *Q* and situated perpendicular to the line of sight between *P* and *Q*. Angle *θ* is measured, then the distance *d* can be determined.

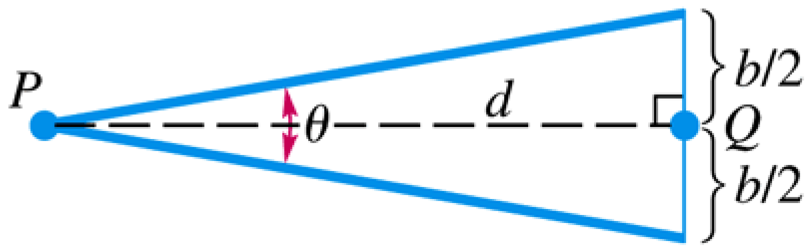


(a) Find *d* with $\theta = 1\ 23'12''$ and $b = 2.0000$ cm.

From the figure, we have $\cot\dfrac{\theta}{2} = \dfrac{d}{b/2} \Rightarrow d = \dfrac{b}{2}\cot\dfrac{\theta}{2}.$

▶ Example 3 USING TRIGONOMETRY TO MEASURE A DISTANCE (continued)

A method that surveyors use to determine a small distance d between two points P and Q is called the **subtense bar method**. The subtense bar with length b is centered at Q and situated perpendicular to the line of sight between P and Q. Angle θ is measured, then the distance d can be determined.



(a) Find d with $\theta = 1\ 23'12''$ and $b = 2.0000$ cm.

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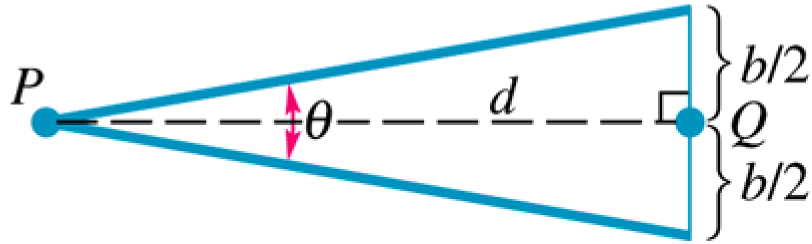
▶ Example 3

USING TRIGONOMETRY TO MEASURE A DISTANCE (continued)



Convert θ to decimal degrees:

$$\theta = 1°23'12'' \approx 1.386667°$$

$$d = \frac{b}{2}\cot\frac{\theta}{2}$$

$$d = \frac{2}{2}\cot\frac{1.386667°}{2}$$

$$\approx 82.634110 \text{ cm}$$

▶ Example 3 USING TRIGONOMETRY TO MEASURE A DISTANCE (continued)

(b) Angle θ usually cannot be measured more accurately than to the nearest 1″. How much change would there be in the value of d if θ were measured 1″ larger?

Since θ is 1″ larger, $\theta = 1\ 23'13'' \approx 1.386944$.

$$d = \frac{2}{2}\cot\frac{1.386944^\circ}{2}$$

$$\approx 82.617558 \text{ cm}$$

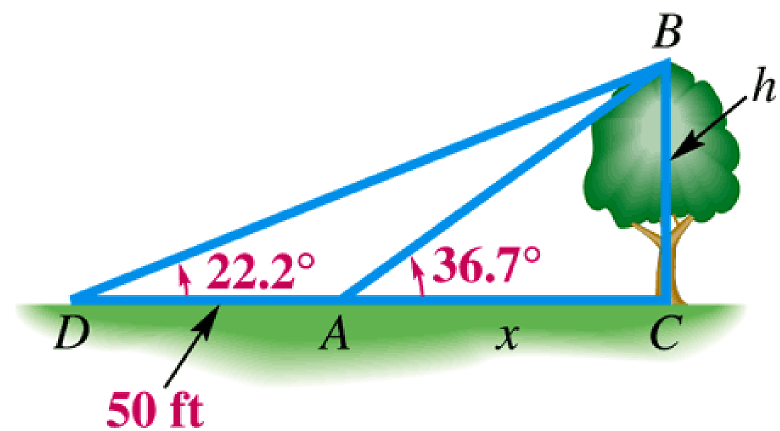
The difference is

$$82.634110 - 82.617558 = .016552 \text{ cm}$$

Example 4 SOLVING A PROBLEM INVOLVING ANGLES OF ELEVATION

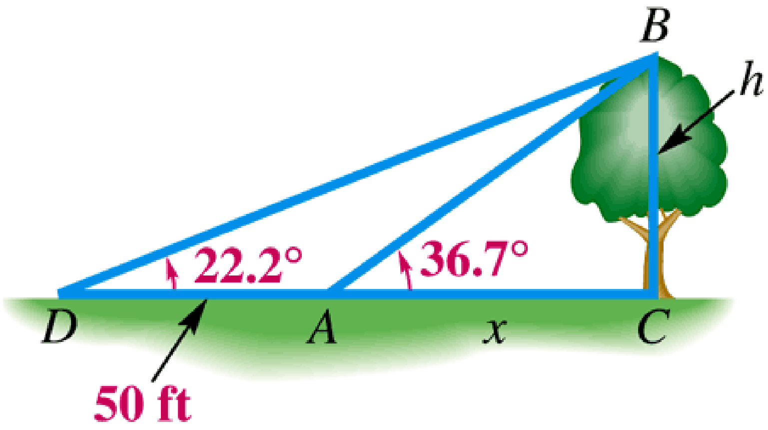
From a given point on the ground, the angle of elevation to the top of a tree is 36.7 . From a second point, 50 feet back, the angle of elevation to the top of the tree is 22.2 . Find the height of the tree to the nearest foot.

The figure shows two unknowns: *x* and *h*.



Since nothing is given about the length of the hypotenuse, of either triangle, use a ratio that does not involve the hypotenuse, tangent.

Example 4 SOLVING A PROBLEM INVOLVING ANGLES OF ELEVATION (continued)



In triangle *ABC*:

$$\tan 36.7° = \frac{h}{x} \Rightarrow h = x \tan 36.7°$$

In triangle *BCD*:

$$\tan 22.2° = \frac{h}{50 + x} \Rightarrow h = (50 + x)\tan 22.2°$$

Each expression equals *h*, so the expressions must be equal.

$$x \tan 36.7° = (50 + x)\tan 22.2°$$

Example 4

SOLVING A PROBLEM INVOLVING ANGLES OF ELEVATION (continued)

$$x\tan 36.7^\circ = (50 + x)\tan 22.2^\circ$$

$$x\tan 36.7^\circ = 50\tan 22.2^\circ + x\tan 22.2^\circ$$

$$x\tan 36.7^\circ - x\tan 22.2^\circ = 50\tan 22.2^\circ$$

$$x(\tan 36.7^\circ - \tan 22.2^\circ) = 50\tan 22.2^\circ$$

$$x = \frac{50\tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

Since $h = x$ tan 36.7 , we can substitute.

$$x = \left(\frac{50\tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}\right)\tan 36.7^\circ$$

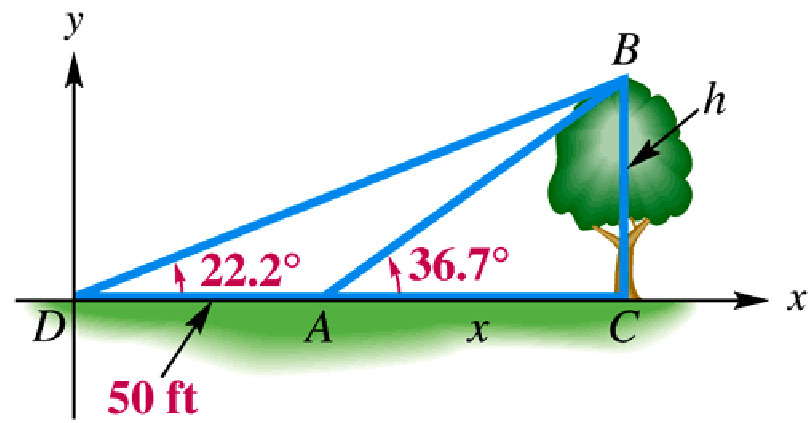
$$\approx 45$$

The tree is about 45 feet tall.

Example 4 SOLVING A PROBLEM INVOLVING ANGLES OF ELEVATION (continued)

Graphing Calculator Solution

Superimpose coordinate axes on the figure with *D* at the origin.



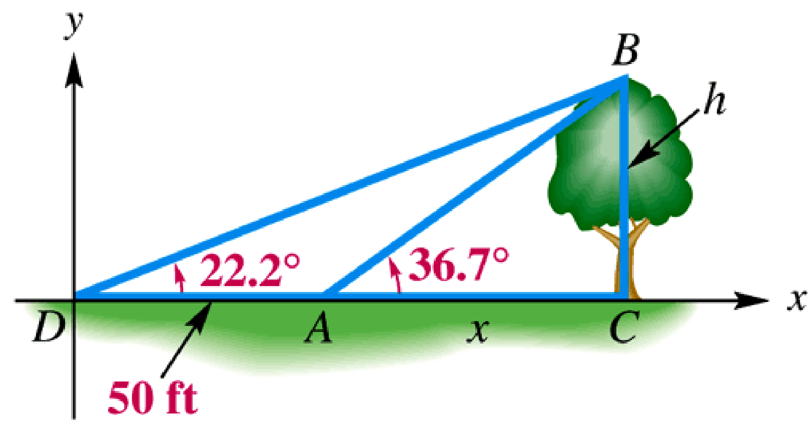
The coordinates of *A* are (50, 0).

The tangent of the angle between the *x*-axis and the graph of a line with equation $y = mx + b$ is the slope of the line. For line *DB*, $m = \tan 22.2$.

Since $b = 0$, the equation of line *DB* is $y_1 = x \tan 22.2°$.

Example 4 SOLVING A PROBLEM INVOLVING ANGLES OF ELEVATION (continued)

The equation of line *AB* is
$y_2 = \tan 36.7° + b.$



Use the coordinates of *A* and the point-slope form to find the equation of *AB*:

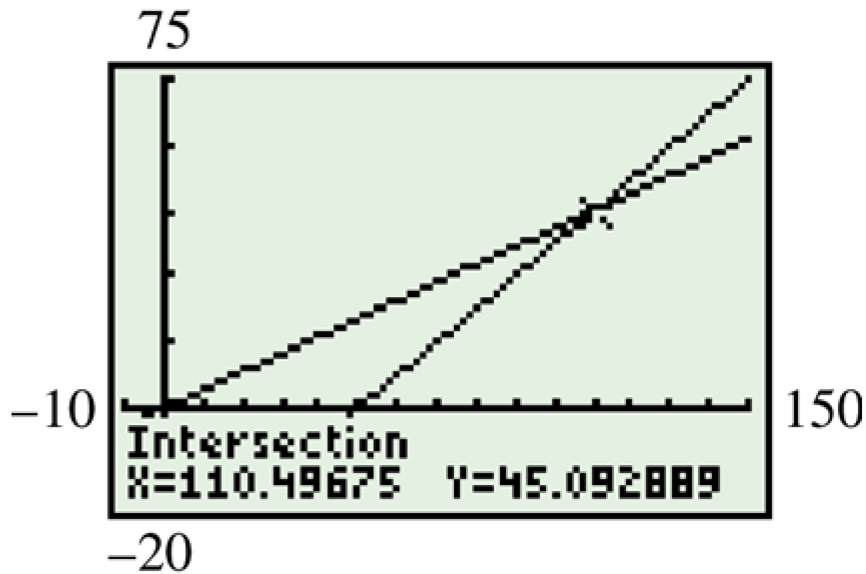
$$y_2 - y_1 = m\left(x_2 - x_1\right)$$

$$y_2 - 0 = m(x - 50) \qquad x_1 = 50,\ y_1 = 0$$

$$y_2 = \tan 36.7°(x - 50)$$

Example 4 SOLVING A PROBLEM INVOLVING ANGLES OF ELEVATION (continued)

Graph y_1 and y_2, then find the point of intersection. The y-coordinate gives the height, h.



The building is about 45 feet tall.