

Section 12.7

Systems of Inequalities

EXAMPLE

Examples of Inequalities in Two Variables

(a) $3x + y \leq 6$

(b) $x^2 + y^2 < 4$

(c) $y^2 > x$

OBJECTIVE 1

- ✓ **1 Graph an Inequality by Hand**

EXAMPLE**Graphing an Inequality**

Graph the linear inequality: $2x - y > 5$

Steps for Graphing an Inequality

STEP 1: Replace the inequality symbol by an equal sign and graph the resulting equation. If the inequality is strict, use dashes; if it is nonstrict, use a solid mark. This graph separates the xy -plane into two or more regions.

STEP 2: In each region, select a test point P .

- (a) If the coordinates of P satisfy the inequality, then so do all the points in that region. Indicate this by shading the region.
- (b) If the coordinates of P do not satisfy the inequality, then none of the points in that region do.

EXAMPLE

Graphing an Inequality

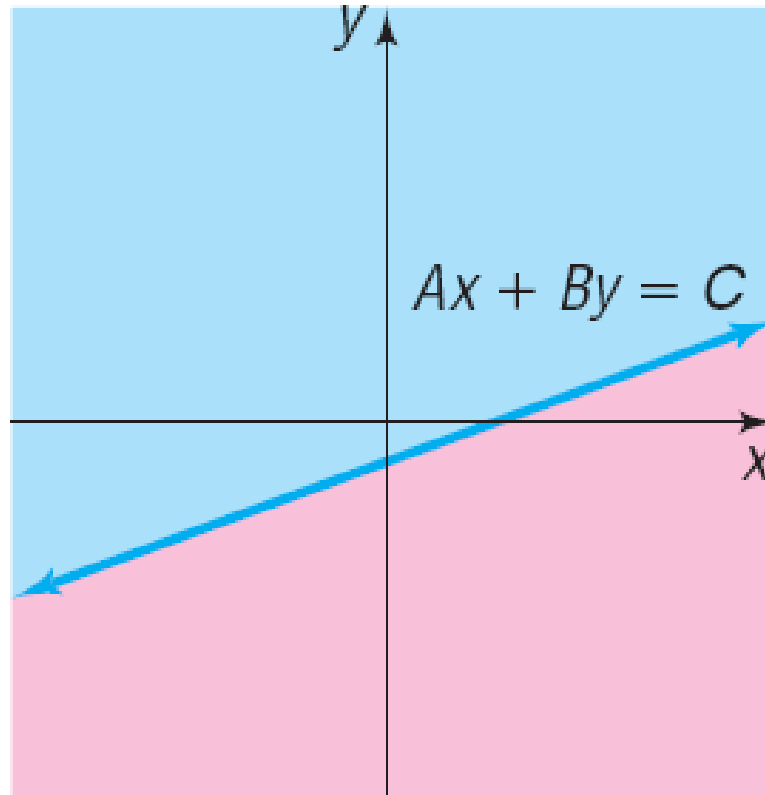
Graph: $y - x^2 < -2$

Linear Inequalities

Linear inequalities are inequalities in one of the forms

$$Ax + By < C \quad Ax + By > C \quad Ax + By \leq C \quad Ax + By \geq C$$

Half-planes



EXAMPLE**Graphing Linear Inequalities by Hand**

Graph: (a) $y > 5$ (b) $y \leq -3x$

OBJECTIVE 2

- ✓ 2 Graph an Inequality Using a Graphing Utility

Steps for Graphing an Inequality Using a Graphing Utility

STEP 1: Replace the inequality symbol by an equal sign and graph the resulting equation. This graph separates the xy -plane into two or more regions.

STEP 2: Select a test point P in each region.

- (a) Use a graphing utility to determine if the test point P satisfies the inequality. If the test point satisfies the inequality, then so do all the points in this region. Indicate this by using the graphing utility to shade the region.
- (b) If the coordinates of P do not satisfy the inequality, then none of the points in that region do.

EXAMPLE

Graphing an Inequality Using a Graphing Utility

Use a graphing utility to graph $-2x + 3y \leq 6$

OBJECTIVE 3

✓ 3 Graph a System of Inequalities

EXAMPLE

Graphing a System of Linear Inequalities by Hand

Graph the system:
$$\begin{cases} 3x + y \leq 5 \\ -2x + y \geq -10 \end{cases}$$

EXAMPLE

Graphing a System of Linear Inequalities Using a Graphing Utility

Graph the system:
$$\begin{cases} 3x + y \leq 5 \\ -2x + y \geq -10 \end{cases}$$

EXAMPLE

Graphing a System of Linear Inequalities by Hand

Graph the system:
$$\begin{cases} x + y \leq 4 \\ x + y \geq 1 \end{cases}$$

EXAMPLE

Graphing a System of Linear Inequalities by Hand

Graph the systems:

$$(a) \begin{cases} x + 2y \geq 4 \\ x + 2y \geq 1 \end{cases}$$

$$(b) \begin{cases} 2x - 2y \geq 5 \\ 2x - 2y \leq 2 \end{cases}$$

EXAMPLE

Graphing a System of Nonlinear Inequalities by Hand

Graph the region below the graph of $2x + y = 4$ and above the graph of $y = x^2 - 2$ by graphing the system:

$$\begin{cases} y \geq x^2 - 9 \\ 2x + y \leq 4 \end{cases}$$

EXAMPLE

Graphing a System of Four Linear Inequalities by Hand

Graph the system:
$$\begin{cases} x + y \leq 10 \\ 2x + 3y \leq 24 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

EXAMPLE**Financial Planning**

A retired couple has up to \$30,000 to invest. As their financial adviser, you recommend that they place at least \$20,000 in Treasury bills yielding 5% and at most \$5000 in corporate bonds yielding 8%.

- (a) Using x to denote the amount of money invested in Treasury bills and y the amount invested in corporate bonds, write a system of linear inequalities that describes the possible amounts of each investment. We shall assume that x and y are in thousands of dollars.
- (b) Graph the system.